Inertial and gravitational effects on electron (muon) g-2 measurements

(Ref, arXiv: 2011.11217)

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Introduction

~ The magnetic moment of an electron ~

Electrons interact with electromagnetic fields through the Dirac equation:

$$i\gamma^\mu \left(\partial_\mu - ieA_\mu\right)\psi = m\psi \quad \longleftarrow \quad \psi = (\phi, \Phi)^T$$

electron (ϕ), positron (Φ)

(γ^μ : gamma matrices , A_μ : vector potential , e : electronic charge , m : electron mass .)

Taking the non-relativistic limit of the electron, we get the Schrodinger equation: $i\partial_0\phi = H\phi$

For the leading order of v/c , Hamiltonian is

$$H = m - eA_0 + \frac{1}{2m}(p_i + eA_i)^2 - \frac{e}{2m}\boldsymbol{S}\cdot\boldsymbol{B}$$

An electron has a magnetic moment

Conventionally, we rewrite it as $-\mu_B \frac{g}{2}\boldsymbol{S}\cdot\boldsymbol{B}$ $\left(\mu_B = \frac{e}{2m}, \quad g = 2\right)$

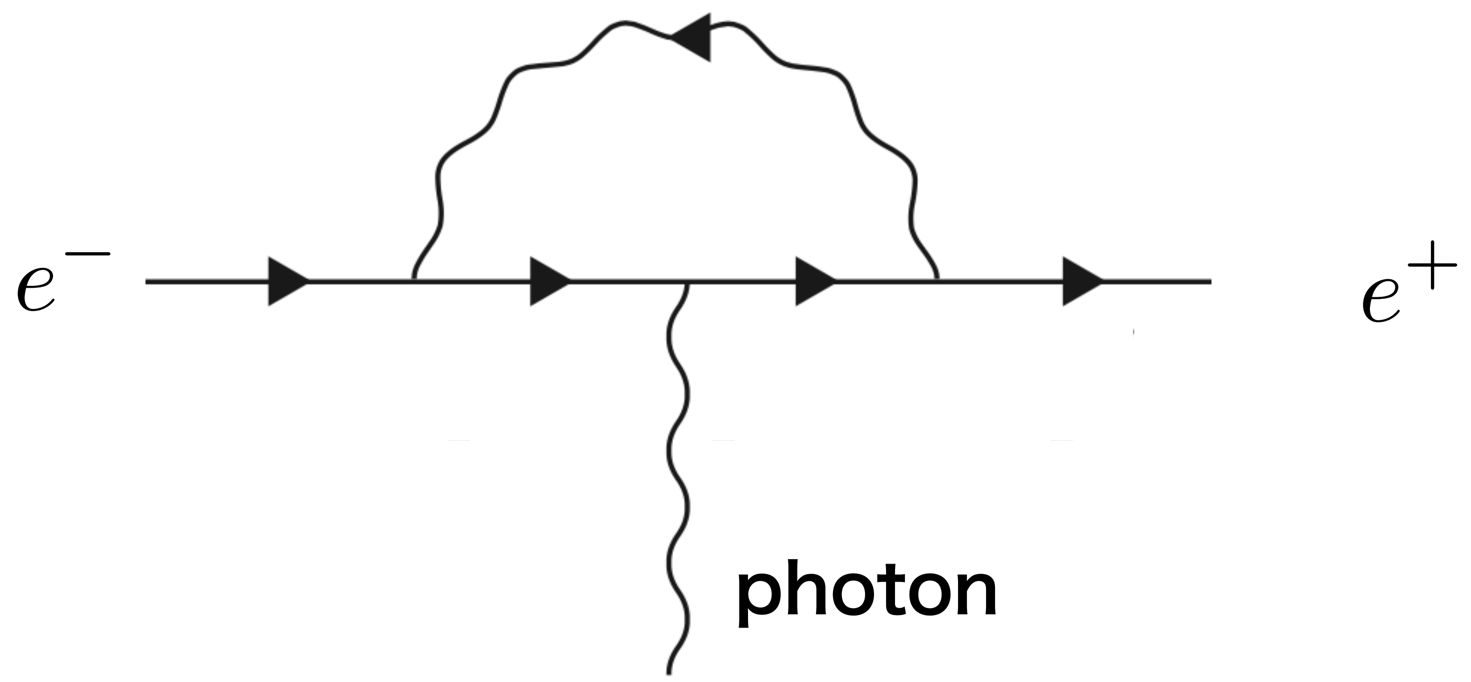
Introduction

~ The magnetic moment of an electron ~

Electrons have a magnetic moment: $H = -\mu_B \frac{g}{2} \boldsymbol{S} \cdot \boldsymbol{B}$

(Bour magneton: $\mu_B = \frac{e}{2m}$, g-factor: g , spin: S , external magnetic field: B)

At the tree level, g-factor is exactly equals to 2. However, It deviates from 2 due to loop corrections



$+ \mathcal{O}(\alpha^2)$ $\left(\alpha = e^2/4\pi \simeq 1/137 \right)$

Now the g-factor has been calculated up to $\mathcal{O}(\alpha^4)$: $\frac{g-2}{2} = a_e^{\mathrm{SM}} = 1{,}159{,}652{,}181.61(23) \times 10^{-12}$

(Hadronic and weak contributions have been included)

(G. Gabrielse, et al, PRL 97, 030802 (2006))

Introduction

~ The magnetic moment of an electron ~

The standard model prediction of the electron g-factor is

$$\frac{g-2}{2} = a_e^{\rm SM} = 1{,}159{,}652{,}181.61(23) \times 10^{-12}$$

(G. Gabrielse, et al, PRL 97, 030802 (2006))

However, it does not coincide with an experimental result:

$$a_e^{\rm exp} = 1{,}159{,}652{,}180.73(28) \times 10^{-12}$$

(D, Hanneke, et al, PRL 100, 120801 (2008))

$$\left(\Delta a_e = a_e^{\rm exp} - a_e^{\rm SM} = -0.88(36) \times 10^{-12} \quad \text{at} \quad 2.5\sigma \right)$$

➡ Implication of a new physics? something has been overlooked?

➡ Effects of Earth's gravity?

(T. Morishima, T. Futamase, H.M. Shimizu (2018), etc...)

※ muon g-factor also has a discrepancy at 4.2σ

Talk plan

- How to measure the electron g-factor?
- Effects of Earth's gravity on a Dirac particle
- Gravitational modifications in electron g-factor measurements
  - correction to the trajectory of the cyclotron motion
  - correction to the spin precession
  - spin-obit coupling

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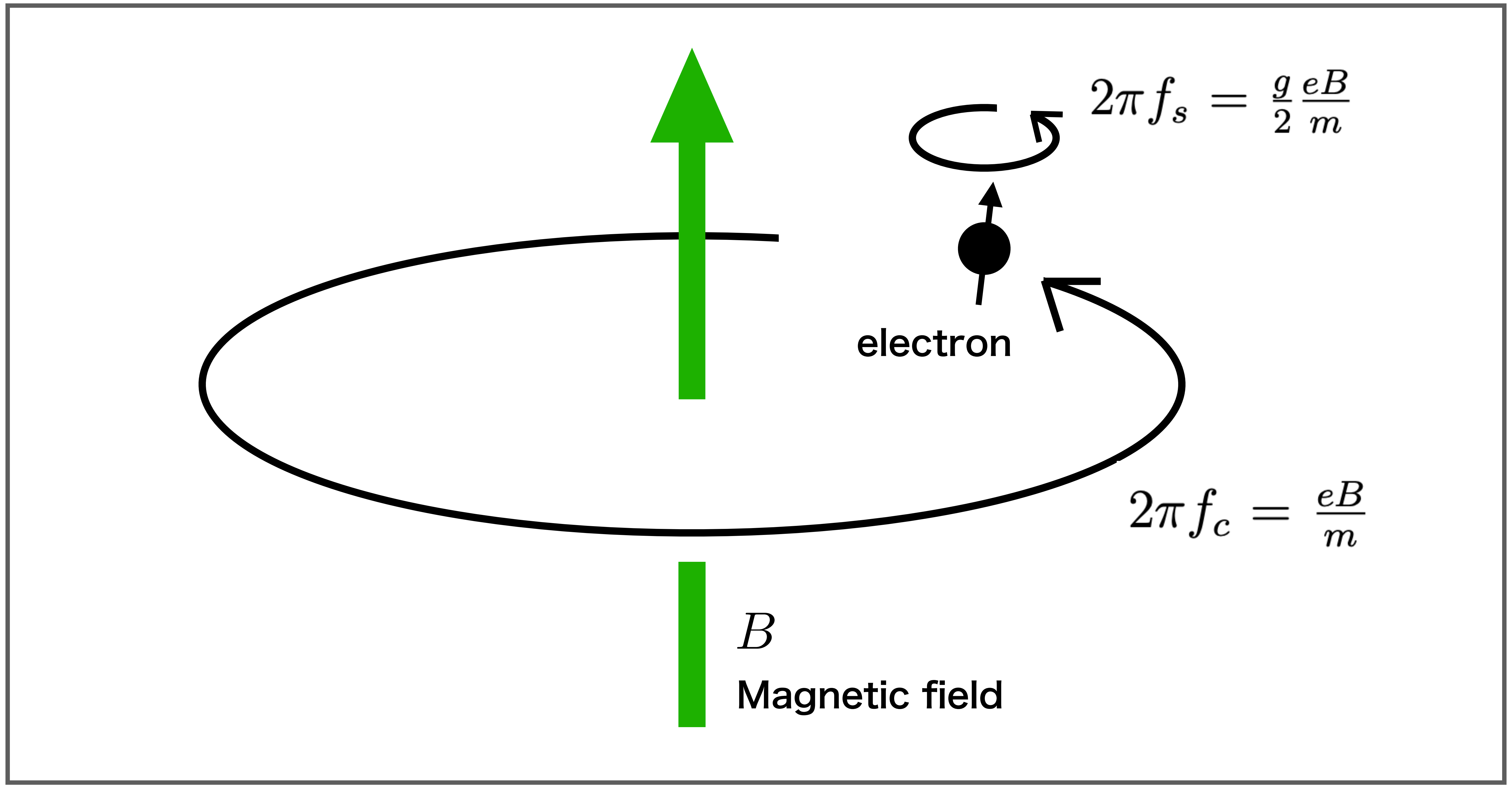
Electron g–factor measurements

An electron experiences the cyclotron motion and the spin precession in the presence of an external magnetic field



Measuring the cyclotron frequency f_c and the spin precession frequency f_s, we can determine the g-factor

$$\frac{g}{2} = \frac{f_s}{f_c}$$

Electron g−factor measurements

The observed g-factor is $$\frac{g}{2} = \frac{f_s}{f_c}$$

We will see that effects of Earth's gravity modify

- the trajectory of the cyclotron motion → modifying f_c
- the spin kinematics of spin precession → modifying f_s

Also a spin-obit coupling will appears due to gravity → modifying f_c & f_s

Then modified g-factor is $$\frac{\delta g}{2} = \frac{f_s + \delta f_s}{f_c + \delta f_c} - \frac{f_s}{f_c}$$

→ Our goal is to calculate δf_s and δf_c due to Earth's gravity

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Dirac equation in curved spacetime

In order to study gravitational effects on an electron, we consider the Dirac equation in curved spacetime

$$i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu\right)\psi = m\psi$$

$\gamma^{\hat{\alpha}}$: gamma matrices , A_μ : vector potential , e : electronic charge ,

m : electron mass , $e^{\mu}_{\hat{\alpha}}$: tetrad which satisfy $e^{\hat{\alpha}}_{\mu} e^{\hat{\beta}}_{\nu} \eta_{\hat{\alpha}\hat{\beta}} = g_{\mu\nu}$,

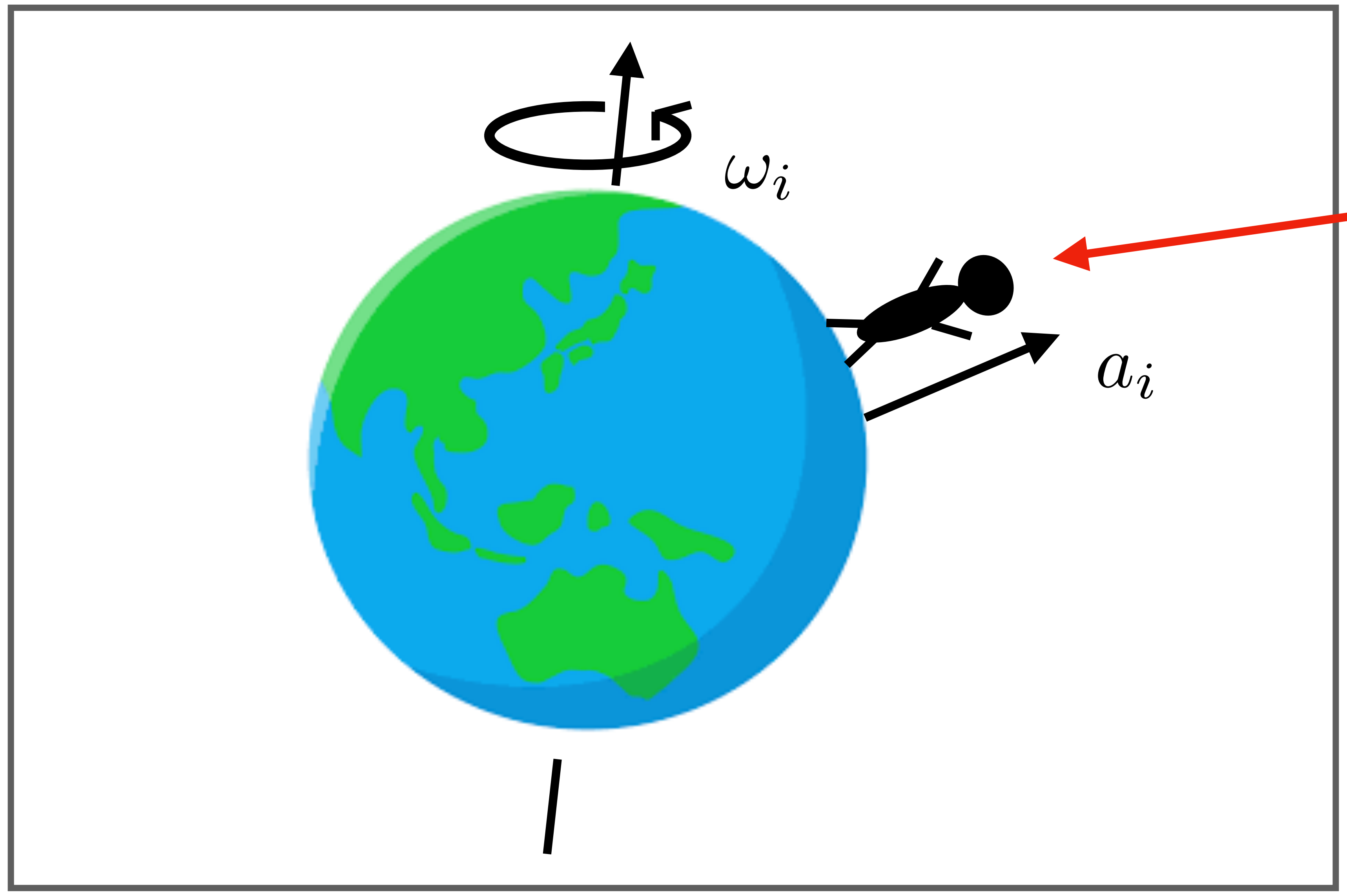
$\Gamma_\mu = \frac{1}{2} e^{\hat{\alpha}}_{\nu} \sigma_{\hat{\alpha}\hat{\beta}} \left(\partial_\mu e^{\nu\hat{\beta}} + \Gamma^{\nu}_{\lambda\mu} e^{\lambda\hat{\beta}}\right)$: spin connection ,

$\sigma_{\hat{\alpha}\hat{\beta}} = \frac{1}{4}[\gamma_{\hat{\alpha}}, \gamma_{\hat{\beta}}]$: generator of the Lorentz group

The Dirac equation satisfies the local Lorentz covariance and the general covariance.

What is the appropriate coordinate (metric) reflecting Earth’s gravity…?

Dirac equation in curved spacetime



This observer is not freely falling due to the Earth

There are three kinds of gravitational effects from the Earth

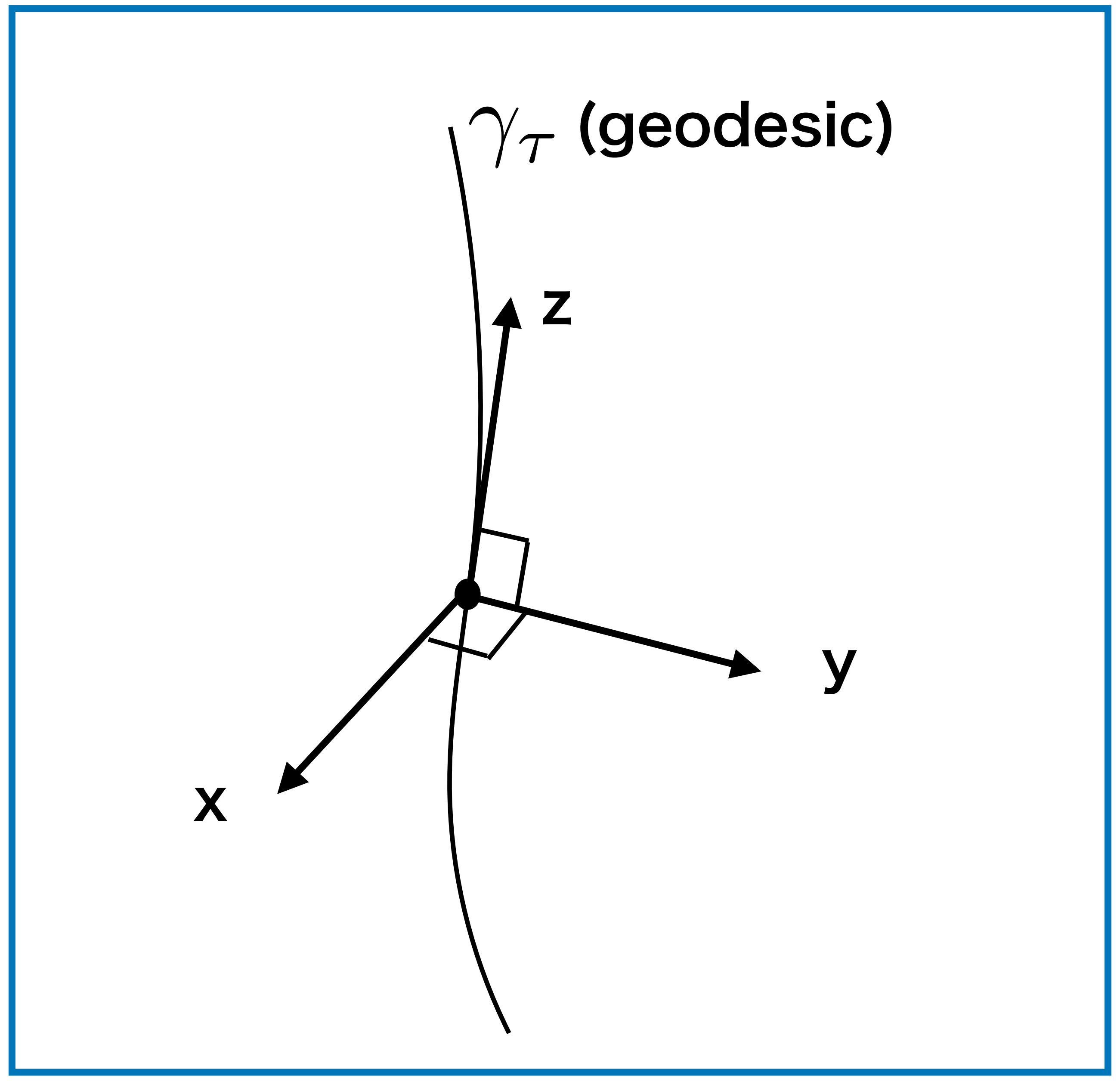
- The linear acceleration a_i
- The rotation ω_i

} Inertial effects due to non-freely falling motion

- The tidal force ← Characterized by the Riemann tensor $R_{\mu\nu\lambda\sigma}$

Fermi normal coordinates

For a freely falling observer,
we can construct a fermi normal coordinate, which move with the geodesic:



The metric for a fermi normal coordinate is given by (up to the linear order)

$$\left(\begin{aligned} g_{00} &= -1 - R_{0i0j}|_{\gamma_\tau} x^i x^j \ , \\ g_{0i} &= -\frac{2}{3} R_{0jik}|_{\gamma_\tau} x^j x^k \ , \\ g_{ij} &= \delta_{ij} - \frac{1}{3} R_{ikjl}|_{\gamma_\tau} x^k x^l \ . \end{aligned} \right.$$

(F.K.Manasse, W.Misner. (1963))

$\longrightarrow$ deviation from the freely falling

$$\left(\begin{aligned} g_{00} &= -1 - 2a_i x^i - R_{0i0j} x^i x^j , \\ g_{0i} &= -\omega_k \epsilon_{0ijk} x^j - \frac{2}{3} R_{0jik} x^j x^k , \\ g_{ij} &= \delta_{ij} - \frac{1}{3} R_{ikjl} x^k x^l , \end{aligned} \right.$$

(W. T. Ni, M. Zimmermann. (1978))

Proper reference frame

An accelerating (a_i) and rotating (ω_i) observer in weak gravitational field ($R_{\mu\nu\lambda\sigma}$) can be characterized by the proper reference coordinate:

(up to leading order)

$$
\begin{cases}
g_{00} = -1 - 2a_i x^i - R_{0i0j} x^i x^j , \\
g_{0i} = -\omega_k \epsilon_{0ijk} x^j - \frac{2}{3} R_{0jik} x^j x^k , \\
g_{ij} = \delta_{ij} - \frac{1}{3} R_{ikjl} x^k x^l ,
\end{cases}
$$

Riemann tensors are evaluated at $\boldsymbol{x} = 0$

(W.-T. Ni, M. Zimmermann, PRD 17, 1473 (1978))

In the case of the Earth's gravity, we specifies

$$
|\boldsymbol{a}| = 9.81\,\mathrm{m/s^2} \ , \quad |\boldsymbol{\omega}| = 7.27 \times 10^{-5}\,\mathrm{rad/s} \ , \quad R_{0i0j} = \left(G\frac{M}{r} \right)_{,ij} \ , \qquad \cdots
$$

Dirac equation in curved spacetime

We got the set up to consider Earth's gravity

We now consider the proper reference frame

$$\begin{cases} g_{00} = -1 - 2a_i x^i - R_{0i0j} x^i x^j, \\ g_{0i} = -\omega_k \epsilon_{0ijk} x^j - \frac{2}{3} R_{0jik} x^j x^k, \\ g_{ij} = \delta_{ij} - \frac{1}{3} R_{ikjl} x^k x^l, \end{cases}$$

in the Dirac equation in curved spacetime

$$i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu\right)\psi = m\psi$$

How do we derive the magnetic moment of an electron...?

Dirac equation in curved spacetime

We take the non-relativistic limit of the Dirac equation in order to derive the magnetic moment.

$$i\gamma^{\hat{\alpha}} e^{\mu}_{\hat{\alpha}} \left(\partial_\mu + \Gamma_\mu + ieA_\mu\right)\psi = m\psi$$

rewriting

$$i\gamma^0 \partial_0 \psi = \left[i\gamma^0 \left(\Gamma_0 + ieA_0\right) - i\gamma^j \left(\partial_j - \Gamma_j - ieA_j\right) + m\right]\psi$$
$$= \gamma^0 \underline{H} \psi \ ,$$

Identifying the non-relativistic Hamiltonian

An explicit calculation gives

$$\begin{aligned} H = & -\frac{i}{2}\gamma^{\hat{0}}\gamma^{\hat{i}}\left(a_i + R_{0i0j}x^j\right) - \frac{i}{4}\gamma^{\hat{i}}\gamma^{\hat{j}} R_{0ikj}x^k - \frac{i}{8}\gamma^{\hat{0}}\gamma^{\hat{i}}\gamma^{\hat{j}}\gamma^{\hat{k}} R_{jkil}x^l - eA_0 \\ & + \left[\gamma^{\hat{0}}\gamma^{\hat{i}}\left(\delta_i^j \left(1 + a_i x^i\right) + \theta_i^j\right) - \gamma^{\hat{i}}\gamma^{\hat{j}}\left(\omega_k \epsilon_{0ilk} x^l + \frac{1}{6}R_{ik0l}x^k x^l\right) + \frac{1}{2}R_{0kjl}x^k x^l\right]\left(-i\partial_j - eA_j\right) \\ & + \left[\gamma^{\hat{0}}\left(1 + a_i x^i + \frac{1}{2}R_{0k0l}x^k x^l\right) - \gamma^{\hat{i}}\left(\omega_k \epsilon_{0ijk} x^j + \frac{1}{6}R_{ik0l}x^k x^l\right)\right] m \ , \end{aligned}$$

Non-relativistic limit of Dirac equation

4×4 Hamiltonian for an electron and a positron

$$H = -\frac{i}{2}\gamma^{\hat{0}}\gamma^{\hat{i}}\left(a_i + R_{0i0j}x^j\right) - \frac{i}{4}\gamma^{\hat{i}}\gamma^{\hat{j}}R_{0ikj}x^k - \frac{i}{8}\gamma^{\hat{0}}\gamma^{\hat{i}}\gamma^{\hat{j}}\gamma^{\hat{k}}R_{jkil}x^l - eA_0$$
$$+\left[\gamma^{\hat{0}}\gamma^{\hat{i}}\left(\delta_i^j\left(1+a_ix^i\right)+\theta_i^j\right) - \gamma^{\hat{i}}\gamma^{\hat{j}}\left(\omega_k\epsilon_{0ilk}x^l + \frac{1}{6}R_{ik0l}x^kx^l\right) + \frac{1}{2}R_{0kjl}x^kx^l\right](-i\partial_j - eA_j)$$
$$+\left[\gamma^{\hat{0}}\left(1+a_ix^i+\frac{1}{2}R_{0k0l}x^kx^l\right) - \gamma^{\hat{i}}\left(\omega_k\epsilon_{0ijk}x^j + \frac{1}{6}R_{ik0l}x^kx^l\right)\right]m\ ,$$

(• **separate an electron and a positron (block diagonalizing)** $\psi = (\phi, \Phi)^T$

• **take the non-relativistic limit (neglecting higher order of** $\mathcal{O}(\frac{1}{m})$ **)**)

(corresponding to v/c & $1/mx$ expansion)

$$H''' = \left(1+a_ix^i+\frac{1}{2}R_{0k0l}x^kx^l\right)m - eA_0 - \omega_k\epsilon_{0ijk}x^i\Pi_j - \omega_iS^i$$
$$+\frac{1}{2m}\left[\delta_{ij}\left(1+a_kx^k+\frac{1}{2}R_{0k0l}x^kx^l\right)+\frac{1}{3}R_{jkil}x^kx^l\right]\Pi_i\Pi_j$$
$$-\frac{e}{m}S^iB^j\left[\delta_{ij}\left(1+a_kx^k+\frac{1}{2}R_{0k0l}x^kx^l+\frac{1}{6}R_{mkml}x^kx^l\right)-\frac{1}{6}R_{ikjl}x^kx^l\right]$$
$$+\frac{1}{2}\epsilon_{0ijl}S^lR_{ijk0}x^k + \frac{1}{2m}\epsilon_{0ijk}a^i\Pi^jS^k + \frac{1}{4m}\epsilon_{0ijk}S^k\left(R_{ijlm}+2\delta_{jm}R_{0i0l}\right)x^l\Pi_m$$

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Effects of Earth's gravity on g-factor

We have the Hamiltonian for a non-relativistic electron

$$H''' = \left(1 + a_i x^i + \frac{1}{2} R_{0k0l} x^k x^l\right) m - eA_0 - \omega_k \epsilon_{0ijk} x^i \Pi_j - \omega_i S^i$$
$$+ \frac{1}{2m}\left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2} R_{0k0l} x^k x^l\right) + \frac{1}{3} R_{jkil} x^k x^l\right] \Pi_i \Pi_j$$
$$- \frac{e}{m} S^i B^j \left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2} R_{0k0l} x^k x^l + \frac{1}{6} R_{mkml} x^k x^l\right) - \frac{1}{6} R_{ikjl} x^k x^l\right]$$
$$+ \frac{1}{2} \epsilon_{0ijl} S^l R_{ijk0} x^k + \frac{1}{2m} \epsilon_{0ijk} a^i \Pi^j S^k + \frac{1}{4m} \epsilon_{0ijk} S^k \left(R_{ijlm} + 2\delta_{jm} R_{0i0l}\right) x^l \Pi_m$$

Kinetic terms

Spin interactions

Spin-orbit couplings

$$\left(\Pi_j = -i\partial_j - eA_j\right)$$

We will see Earth's gravity affects

- the trajectory of the cyclotron motion
- the spin kinematics of spin precession
- a spin-obit coupling appears

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Particle trajectory

The Hamiltonian concerned with cyclotron motion is

$$H_{\rm orbit} = \left(1 + a_i x^i + \frac{1}{2}R_{0k0l}x^k x^l\right) m - eA_0 - \omega_k \epsilon_{0ijk} x^i \left(p_j - eA_j\right) + \frac{i}{2m} a_i \left(p_i - eA_i\right)$$
$$+\frac{1}{2m}\left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2}R_{0k0l}x^k x^l\right) + \frac{1}{3}R_{jkil}x^k x^l\right]\left(p_i - eA_i\right)\left(p_j - eA_j\right) \ ,$$

Equation of motion

$$\ddot{x}^i = -a^i - R_{0i0j}x^j + \left[\left\{\delta_{ij}\left(1 + a_m x^m + \frac{1}{2}R_{0m0n}x^m x^n\right) + \frac{1}{3}R_{imjn}x^m x^n\right\}\frac{e}{m}B^l + 2\delta_{ij}\omega^l\right]\epsilon_{0jkl}\dot{x}^k \ ,$$

solving "exactly"

giving rise to the cyclotron motion x^i_{cy} with $2\pi f_c = eB/m$

solving perturbatively

$$\delta f_c \simeq f_c \left(a_i x^i_{cy} + \frac{2\omega}{2\pi f_c}\cos\theta\right)$$

(※ $R_{0i0i}x^i_{cy}x^j_{cy} \sim G\frac{M}{x_0^3}x^2_{cy} \ll a_i x^i_{cy} \sim G\frac{M}{x_0^2}x_{cy}$

x_0 : radius of the Earth, x^i_{cy} : cyclotron orbit

θ : angle between ω and B)

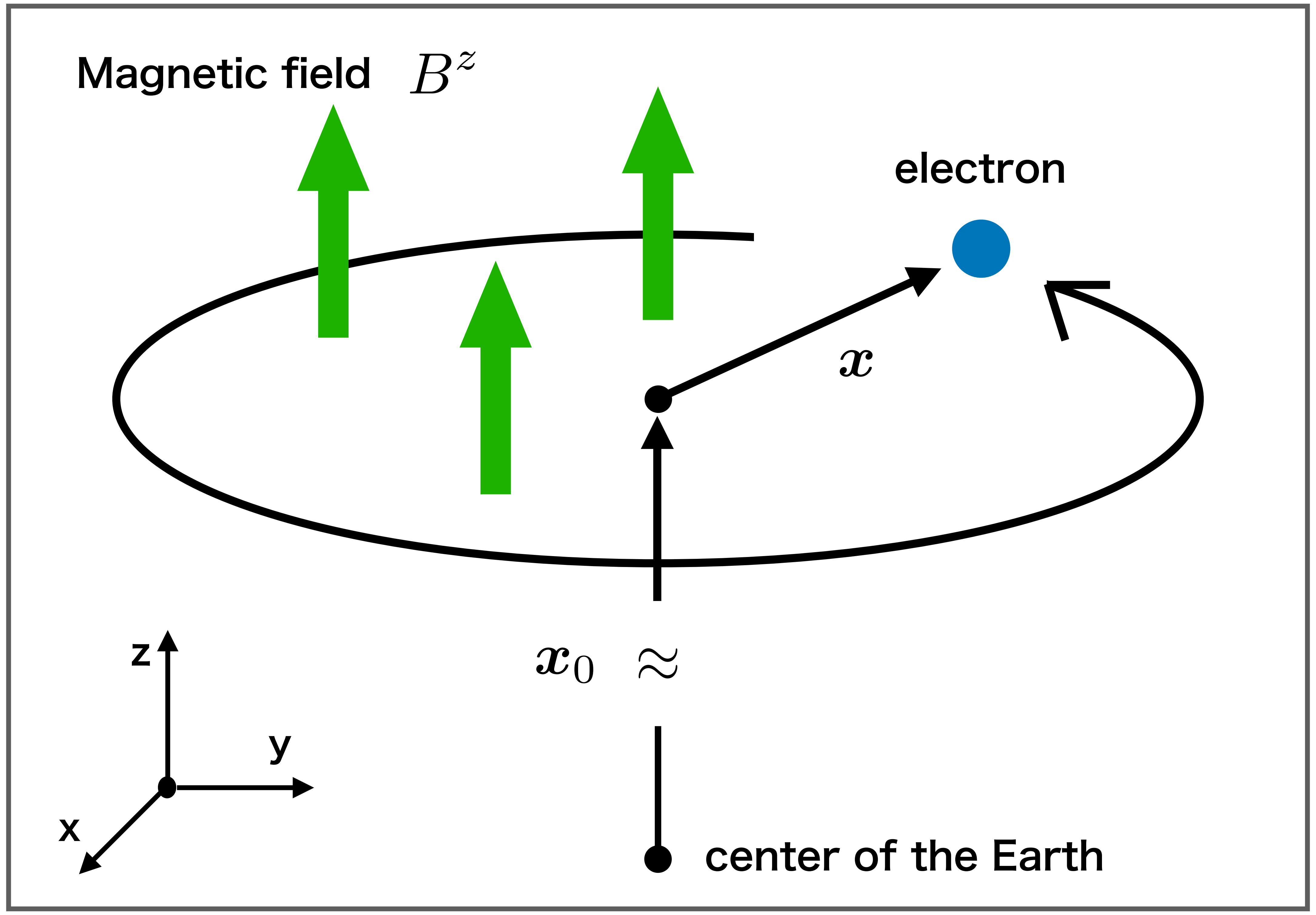
Particle trajectory

We consider the equation

$$\ddot{x}^i = -a^i - R_{0i0j}x^j + \frac{e}{m}B^l\epsilon_{0ikl}\dot{x}^k$$

with an experimental setting $\boldsymbol{x}_0 = (0,0,z_0)$, $\boldsymbol{B} = (0,0,B^z)$.

$$\Rightarrow \begin{cases} \ddot{x} = -G\dfrac{M}{x_0^3}x + \dfrac{eB^z}{m}\dot{y} \ , \\ \ddot{y} = -G\dfrac{M}{x_0^3}y - \dfrac{eB^z}{m}\dot{x} \ , \\ \ddot{z} = -a^z + 2G\dfrac{M}{x_0^3}z \ . \end{cases}$$



Particle trajectory

The equations of motion in x-y plane

$$\ddot{x} = -G\frac{M}{x_0^3}x + \frac{eB^z}{m}\dot{y} \ ,$$
$$\ddot{y} = -G\frac{M}{x_0^3}y - \frac{eB^z}{m}\dot{x} \ ,$$

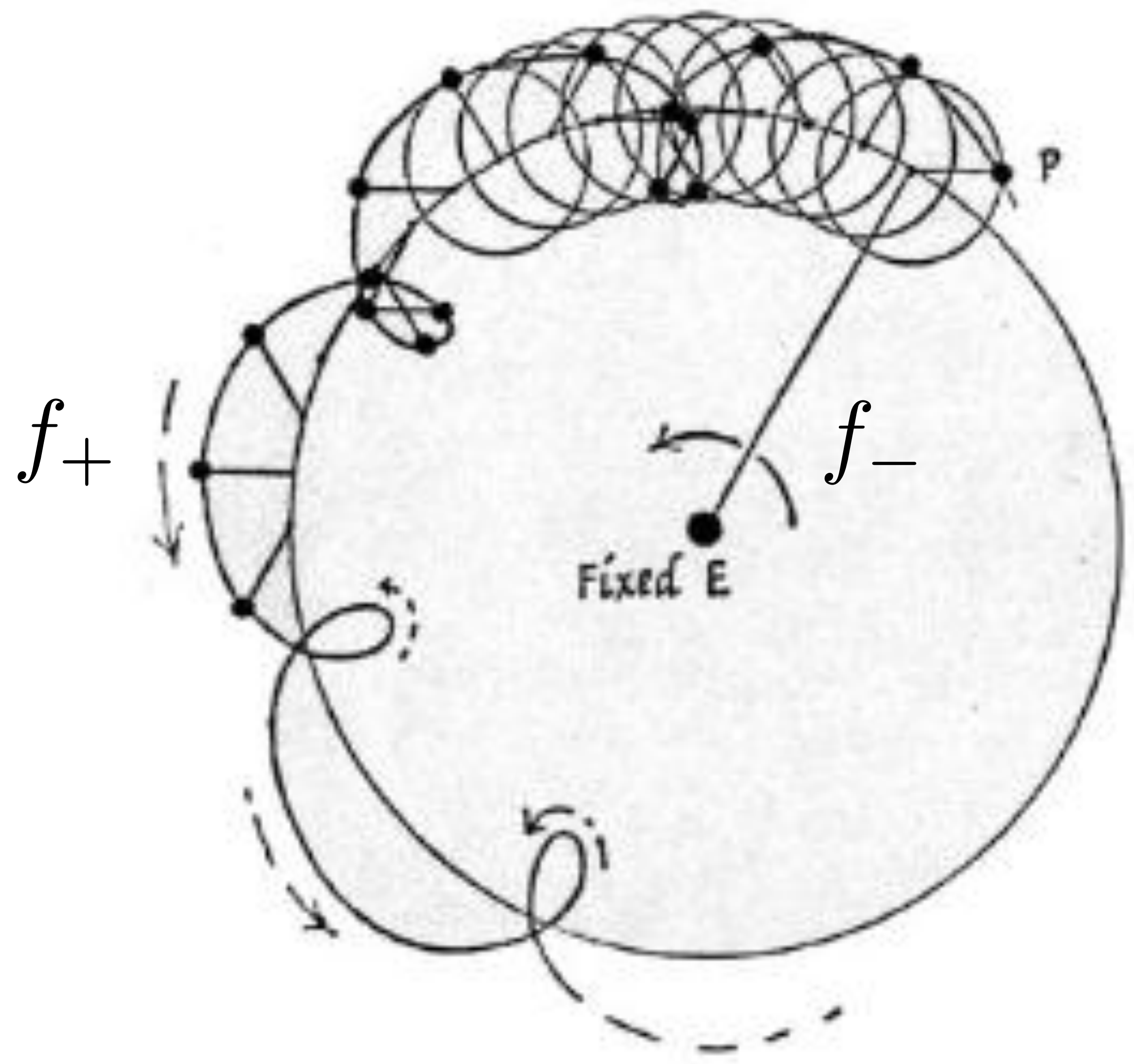
can be solved exactly as

$$\begin{cases} x = C_1\cos(-2\pi f_+ t) + C_2\cos(-2\pi f_- t) \ , \\ y = C_1\sin(-2\pi f_+ t) + C_2\sin(-2\pi f_- t) \ , \end{cases}$$

where

$$2\pi f_\pm = \frac{2\pi f_c \pm \sqrt{(2\pi f_c)^2 + 4GM/x_0^3}}{2}$$

→ epicyclic orbit



The modified cyclotron frequency is

$$f_+ \simeq f_c + \frac{GM/x_0^3}{(2\pi f_c)^2}$$

Particle trajectory

$$\ddot{x}^i = -a^i - R_{0i0j}x^j + \left[\left\{\delta_{ij}\left(1 + a_m x^m + \frac{1}{2}R_{0m0n}x^m x^n\right) + \frac{1}{3}R_{imjn}x^m x^n\right\}\frac{e}{m}B^l + 2\delta_{ij}\omega^l\right]\epsilon_{0jkl}\dot{x}^k \,,$$

solving "exactly"

$$f_c + \delta f_c \simeq f_c + \frac{GM/x_0^3}{(2\pi f_c)^2}$$

giving rise to the cyclotron motion with $2\pi f_c = eB/m$

solving perturbatively

$$\delta f_c \simeq f_c\left(a_i x^i_{cy} + \frac{2\omega}{2\pi f_c}\cos\theta\right)$$

Total gravitational correction of the cyclotron frequency is

$$f_c + \delta f_c = f_c\left(1 + a_i x^i_{cy} - \frac{2\omega}{2\pi f_c}\cos\theta + \frac{GM/x_0^3}{(2\pi f_c)^2}\right)$$

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Spin kinematics

The Hamiltonian concerned with spin precession is

$$H_{\text{spin}} = \frac{1}{2}\epsilon_{0ijl}S^l R_{ijk0}x^k - \omega_i S^i \\ -\frac{e}{m}S^i B^j \left[\delta_{ij}\left(1 + a_k x^k + \frac{1}{2}R_{0k0l}x^k x^l + \frac{1}{6}R_{mkml}x^k x^l\right) - \frac{1}{6}R_{ikjl}x^k x^l\right] .$$

Equation of motion is

$$\dot{S}^a = -\epsilon_{0aib}S^b\left[\frac{e}{m}B^i\left(1 + a_k x^k\right) + \omega_i\right]$$

$$2\pi f_s = eB/m \qquad \delta f_s$$

Therefore, the modified spin precession frequency is

$$f_s + \delta f_s = f_s\left(1 + a_i x^i_{cy} - \frac{\omega}{2\pi f_s}\cos\theta\right)$$

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Spin–orbit coupling

The Hamiltonian concerned with spin-orbit couplings is

$$H_{\text{spin}-\text{orbit}} = \frac{1}{2m}\Pi_i\Pi_j - \frac{e}{m}S^i B^j + \frac{1}{2m}\epsilon_{0ijk}a^i\Pi^j S^k + \frac{1}{4m}\epsilon_{0ijk}S^k\left(R_{ijlm} + 2\delta_{jm}R_{0i0l}\right)x^l\Pi_m \ ,$$

We rewrite the Hamiltonian as below:

$$H_{\text{spin}-\text{orbit}} = (2\pi f_c)\left(\alpha^\dagger\alpha + \frac{1}{2}\right) - (2\pi f_s)S^z - \Delta\left(\alpha S_+ + \alpha^\dagger S_-\right) \qquad \left(2\pi f_c = 2\pi f_s = eB/m\right)$$

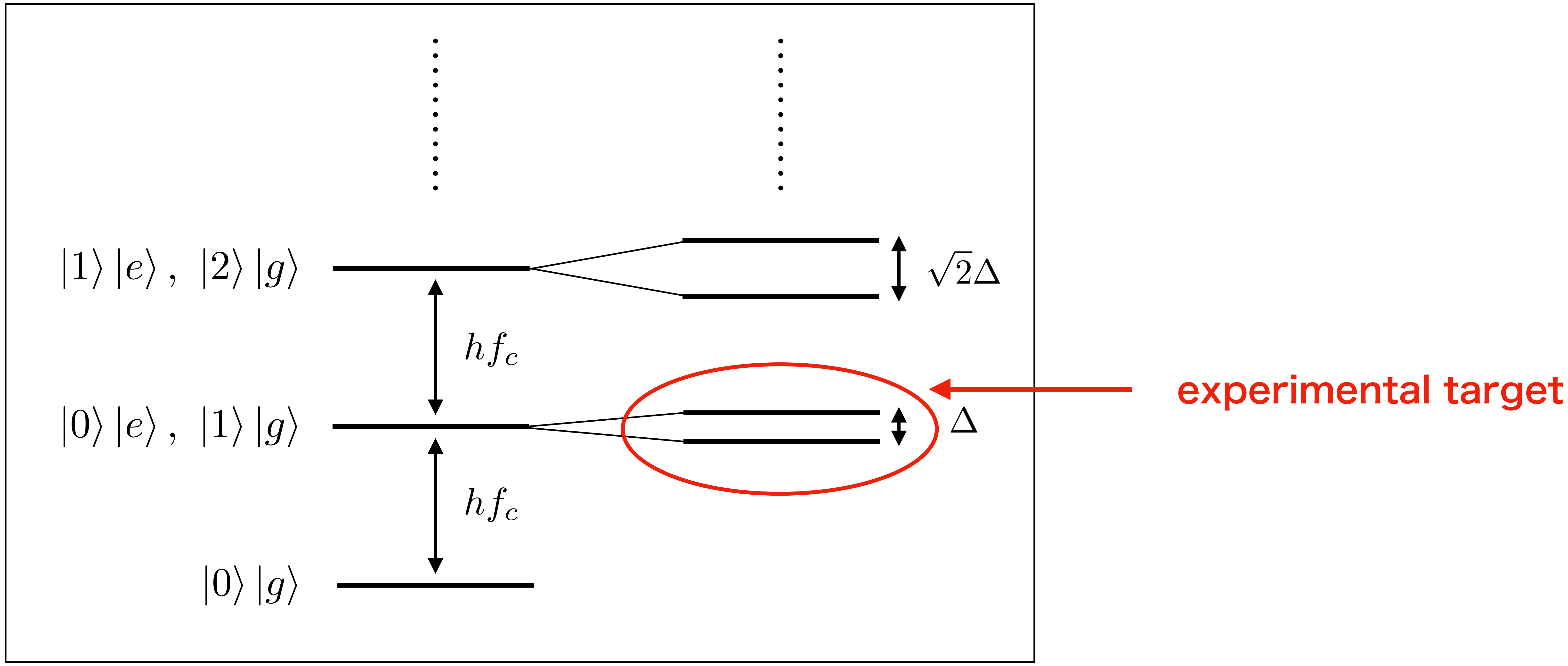
where $\alpha = \frac{1}{\sqrt{2eB^z}}(-i\Pi_x + \Pi_y)$, $\alpha^\dagger = \frac{1}{\sqrt{2eB^z}}(i\Pi_x + \Pi_y)$, are creation and annihilation operators, $\left(\Pi_j = -i\partial_j - eA_j \right)$

$S_+ = S^x - iS^y$, $S_- = S^x + iS^y$ are ladder operators, $\Delta = \sqrt{\frac{2\pi f_s}{8m}}a$ is a coupling constant.

Spin-orbit coupling

$$H_{\text{spin-orbit}} = (2\pi f_c)\left(\alpha^\dagger\alpha + \frac{1}{2}\right) - (2\pi f_s)S^z - \Delta\left(\alpha S_+ + \alpha^\dagger S_-\right) \qquad \left(2\pi f_c = 2\pi f_s = eB/m\right)$$

The spin-orbit coupling breaks the degenerate energy levels between $|n\rangle\,|e\rangle$ and $|n+1\rangle\,|g\rangle$



The dressed frequencies are

$$f_c + \delta f_c = f_c\left(1 - \frac{1}{4\sqrt{2}}\frac{a}{\sqrt{(2\pi f_c)m}}\right), \quad f_s + \delta f_s = f_s\left(1 + \frac{1}{4\sqrt{2}}\frac{a}{\sqrt{(2\pi f_s)m}}\right)$$

Effects of Earth's gravity on g-factor

- modified particle trajectories: $$f_c + \delta f_c = f_c \left(1 + a_i x^i_{cy} - \frac{2\omega}{2\pi f_c}\cos\theta + \frac{GM/x_0^3}{(2\pi f_c)^2}\right)$$
- modified spin kinematics: $$f_s + \delta f_s = f_s \left(1 + a_i x^i_{cy} - \frac{\omega}{2\pi f_s}\cos\theta\right)$$
- a spin-obit coupling: $$f_c + \delta f_c = f_c \left(1 - \frac{1}{4\sqrt{2}}\frac{a}{\sqrt{(2\pi f_c)m}}\right), \quad f_s + \delta f_s = f_s \left(1 + \frac{1}{4\sqrt{2}}\frac{a}{\sqrt{(2\pi f_s)m}}\right)$$

Total correction is

$$\left(\begin{aligned} f_c + \delta f_c &= f_c \left(1 + \cancel{a_i x^i_{cy}} \pm \frac{2\omega}{2\pi f_c}\cos\theta - \frac{1}{4\sqrt{2}}\frac{a}{\sqrt{(2\pi f_c)m}} + \frac{GM/x_0^3}{2(2\pi f_c)^2}\right), \\ f_s + \delta f_s &= f_s \left(1 + \cancel{a_i x^i_{cy}} \pm \frac{\omega}{2\pi f_s}\cos\theta + \frac{1}{4\sqrt{2}}\frac{a}{\sqrt{(2\pi f_c)m}}\right). \end{aligned}\right.$$

Gravitational correction to g-factor is →

$$\frac{\delta g}{2} = \frac{f_s + \delta f_s}{f_c + \delta f_c} - \frac{f_s}{f_c} \simeq \mp \frac{\omega}{2\pi f_c}\cos\theta + \frac{1}{2\sqrt{2}}\frac{a}{\sqrt{(2\pi f_c)m}} - \frac{GM/x_0^3}{(2\pi f_c)^2}$$

Effects of Earth's gravity on g-factor

The most accurate electron g-factor measurement was operated at Harvard Univ. in 2008.

(D, Hanneke, et al, PRL 100, 120801 (2008))

Using the experimental values $f_c = f_s = eB/m \simeq 150\,\mathrm{GHz}$, $\theta \simeq 0.674\,\mathrm{rad}$,

one can estimate the each gravitational correction:

Effects of Earth's rotation	$\frac{\omega}{2\pi f_c}\cos\theta$	5.2×10^{-17}
Spin-orbit coupling through a_i	$\frac{1}{2\sqrt{2}}\frac{a}{\sqrt{(2\pi f_c)m}}$	4.3×10^{-25}
Tidal effect	$-\frac{GM/x_0^3}{(2\pi f_c)^2}$	-1.7×10^{-30}
Sensitivity in the experiment (G. Gabrielse, et al, PRL 97, 030802 (2006))		2.8×10^{-13}

Summary

- The electron g-factor has been measured intensively to confirm the standard model prediction
- There is a discrepancy between an experimental result and the theoretical prediction at 2.5σ
- We probed the possibility that the discrepancy could be explained by effects of Earth's gravity

→ The leading correction comes from the rotation of the Earth, that is $\delta g/2 \simeq 5.2 \times 10^{-17}$

- Although the gravitational effect is smaller than the current sensitivity $\delta g/2 = 2.8 \times 10^{-13}$, it might be detectable in the future

→ improving by 3 digits is possible…?

(G. Gabrielse, et al, arXiv:1904.06174 (2019), X. Fan, et al, arXiv:2011.08136 (2020))

(※ muon sensitivity: $\delta g/2 = 4.6 \times 10^{-7}$ modification from the rotation of the Earth: $\delta g/2 \sim 10^{-12}$

(B. Abi, et al, arXiv:2104.03281 (2021)))