Colloquium @ Theoretical Astrophysics Group in Kyushu University (online), 30th July 2021

Overview of vacuum decay with gravity

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RIKEN

iTHEMS
RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program

I. Introduction

II. Lightning review of vacuum decay

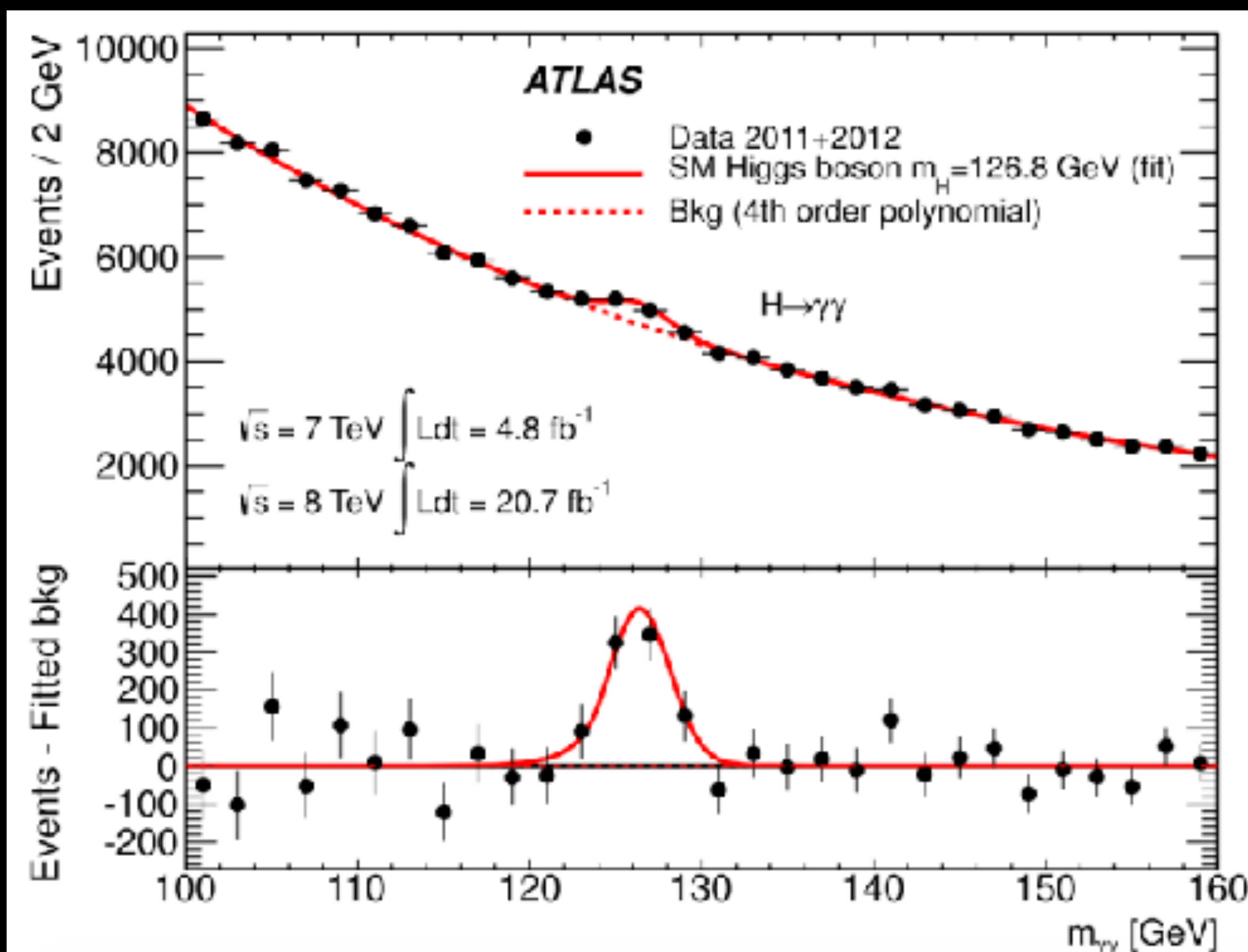
III. Vacuum decay with a black hole

Vacuum decay

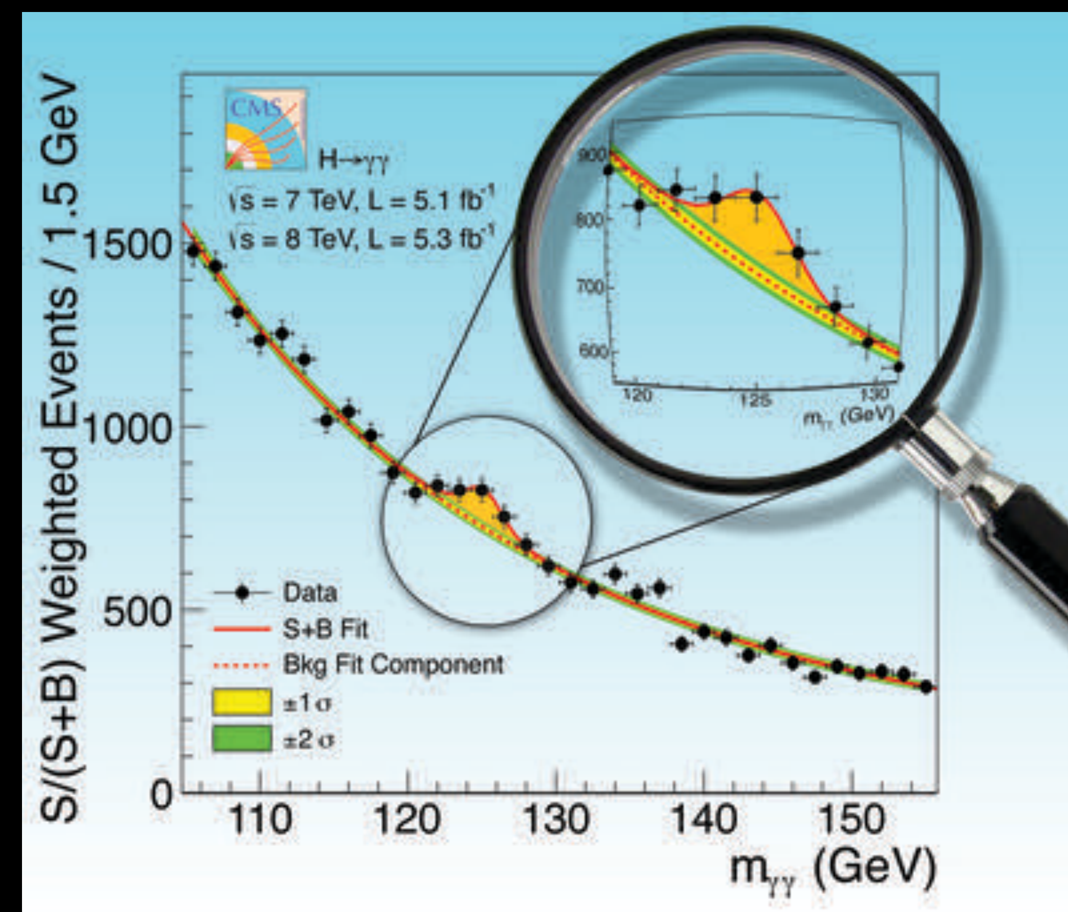
- One of the most important non-perturbative phenomena in the Universe
- Inflation involving vacuum decay -> bubble collisions -> GW background
- Higgs metastability
- Eternal inflation and multiverse
- Vacuum decay seeded by black holes -> constraints on PBH parameters or the parameters of Higgs potential

etc…

A Higgs particle



ATLAS collaboration (2012)

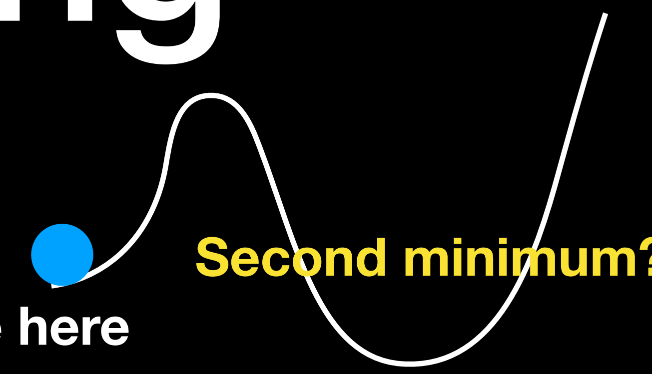


https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIG

A Higgs particle with ~125 GeV has been found!!

Running coupling

$$V = \frac{1}{4}\lambda_{\rm eff}(\phi)\phi^4$$



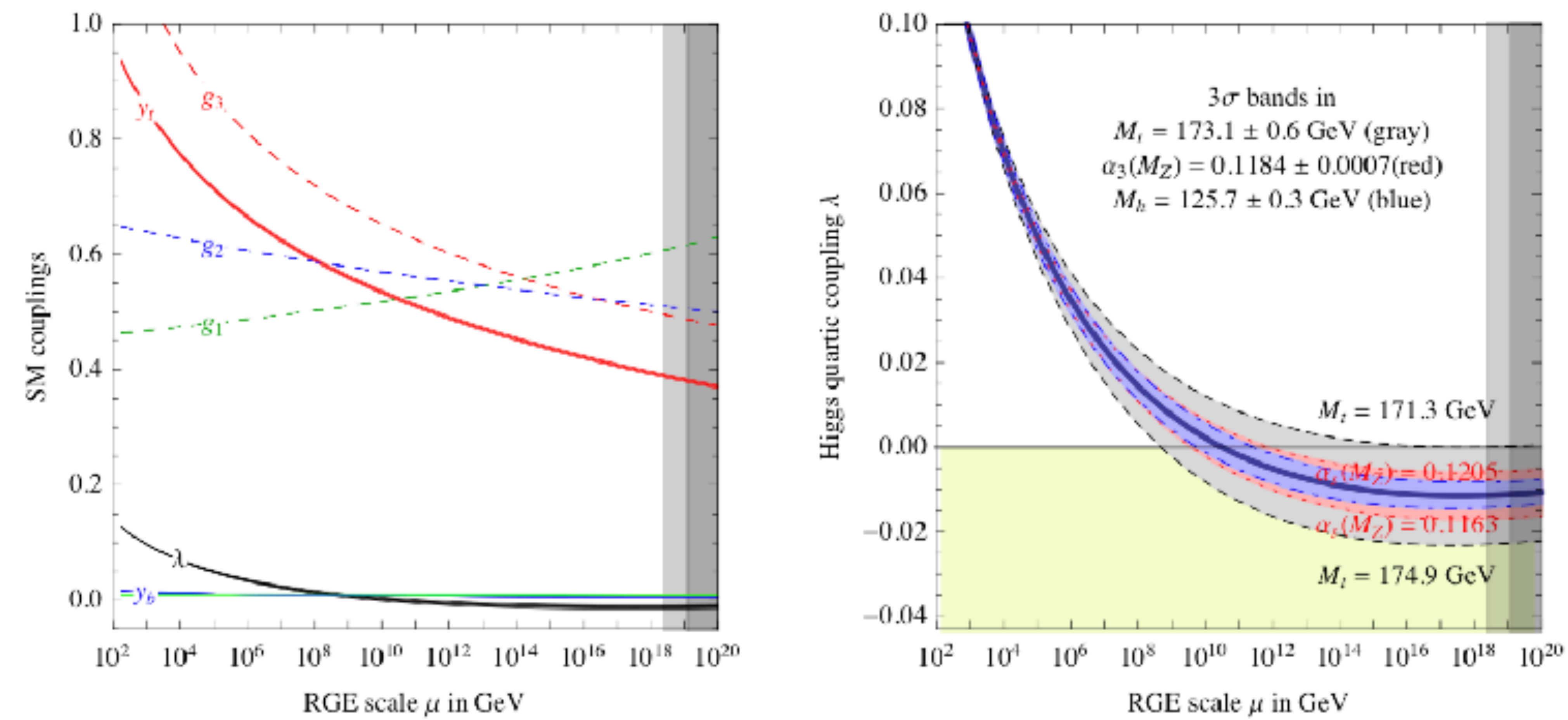
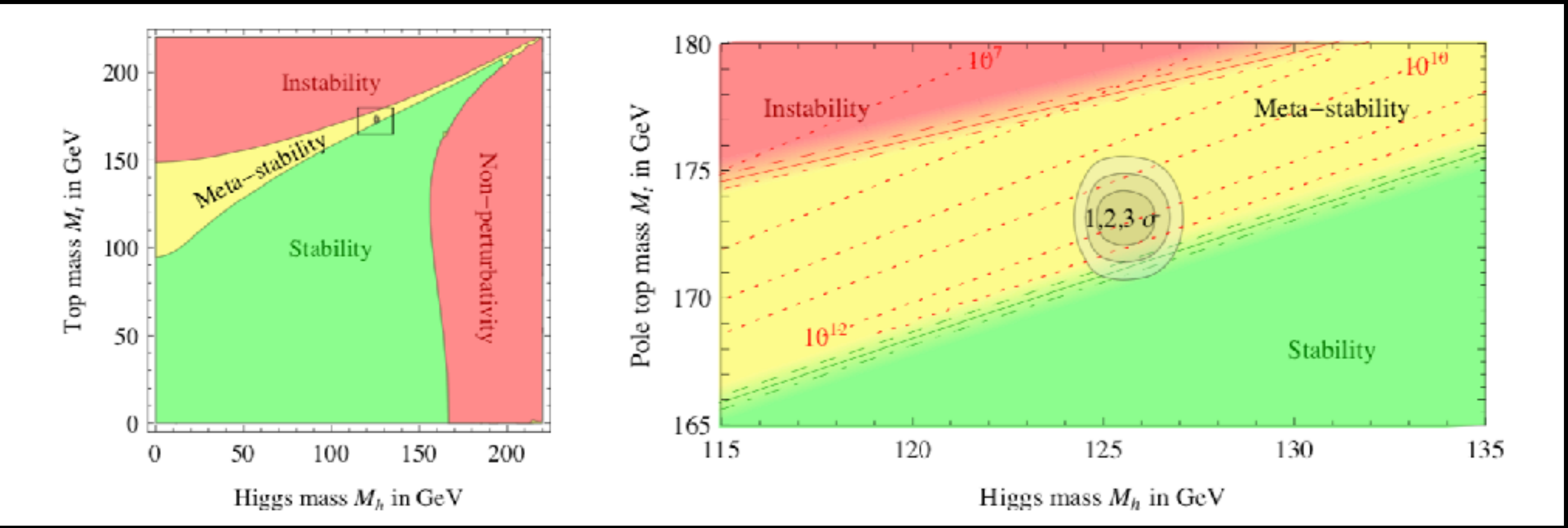


Figure 1: **Left**: *SM RG evolution of the gauge couplings* $g_1 = \sqrt{5/3}g'$, $g_2 = g$, $g_3 = g_s$, *of the top and bottom Yukawa couplings* (y_t, y_b), *and of the Higgs quartic coupling* λ. *All couplings are defined in the* $\overline{\rm MS}$ *scheme. The thickness indicates the* $\pm 1\sigma$ *uncertainty.* **Right**: *RG evolution of* λ *varying* M_t, M_h *and* α_s *by* $\pm 3\sigma$.

Degrassi et al. (2013)

The Higgs self-coupling can be negative at high energies.

Higgs metastability?



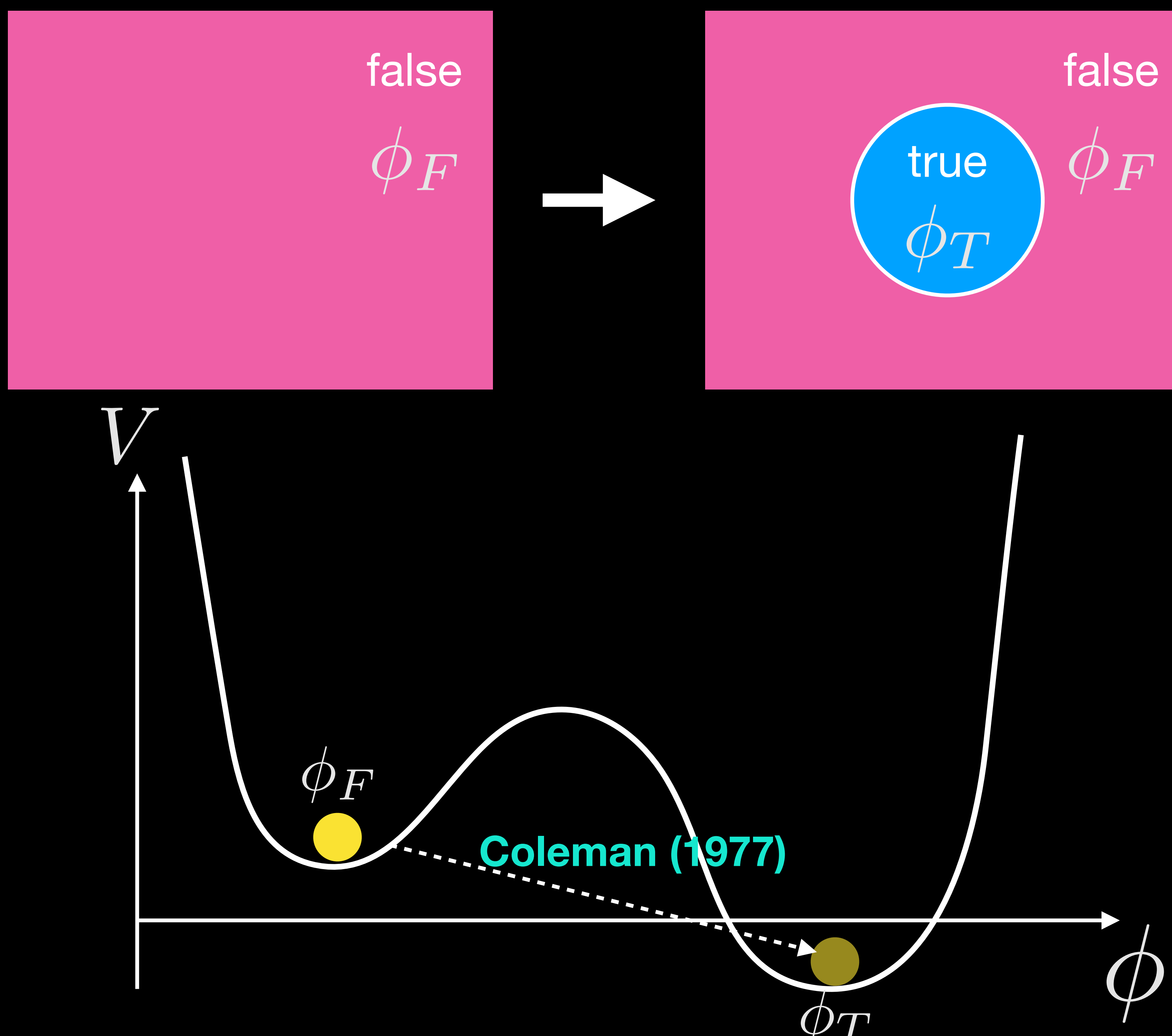
Our vacuum might be in a meta-stable state. Degrassi et al. (2013)

Computation of vacuum decay rate indicates that the life-time of the Universe is many order of magnitude greater than the cosmic age.

Chigusa, Moroi, Shoji (2018)

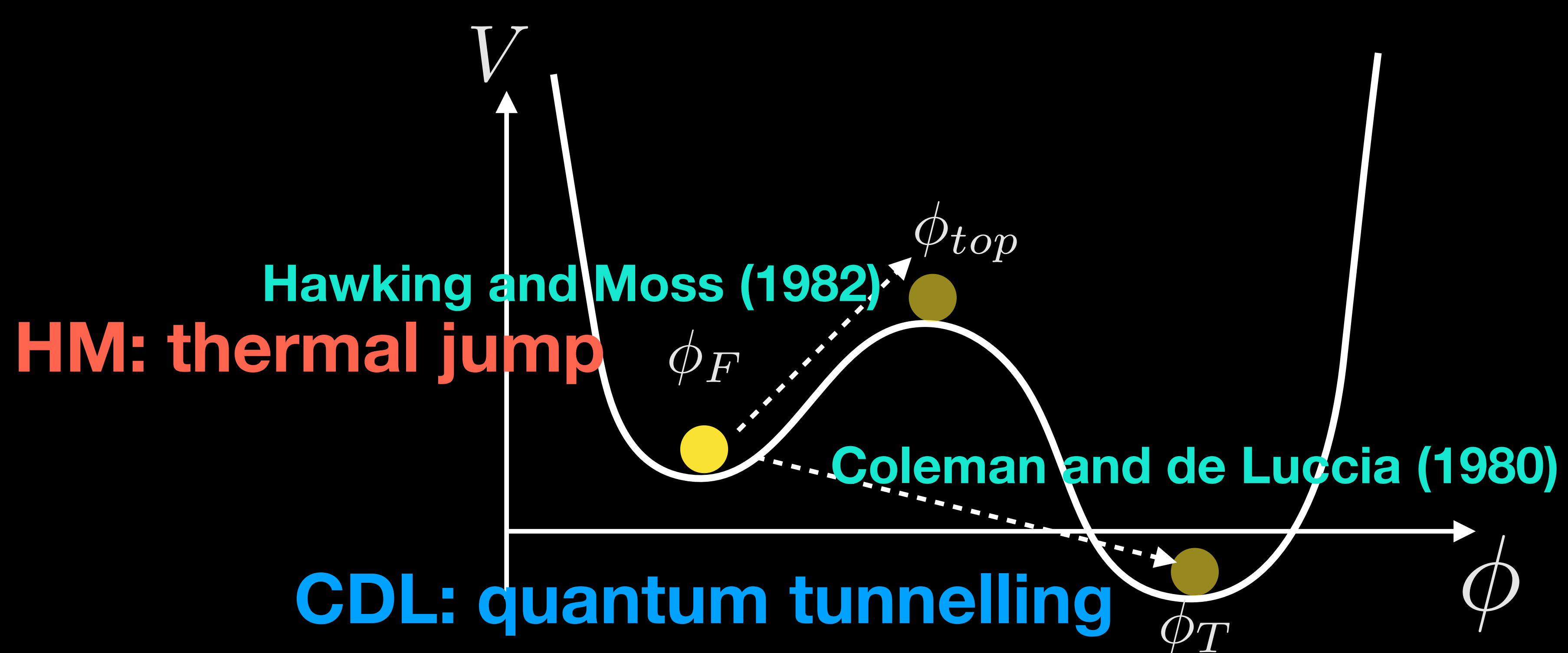
Vacuum phase transition without gravity

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$



Vacuum phase transition with gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$



The absolute value of vacuum energy density is important!!

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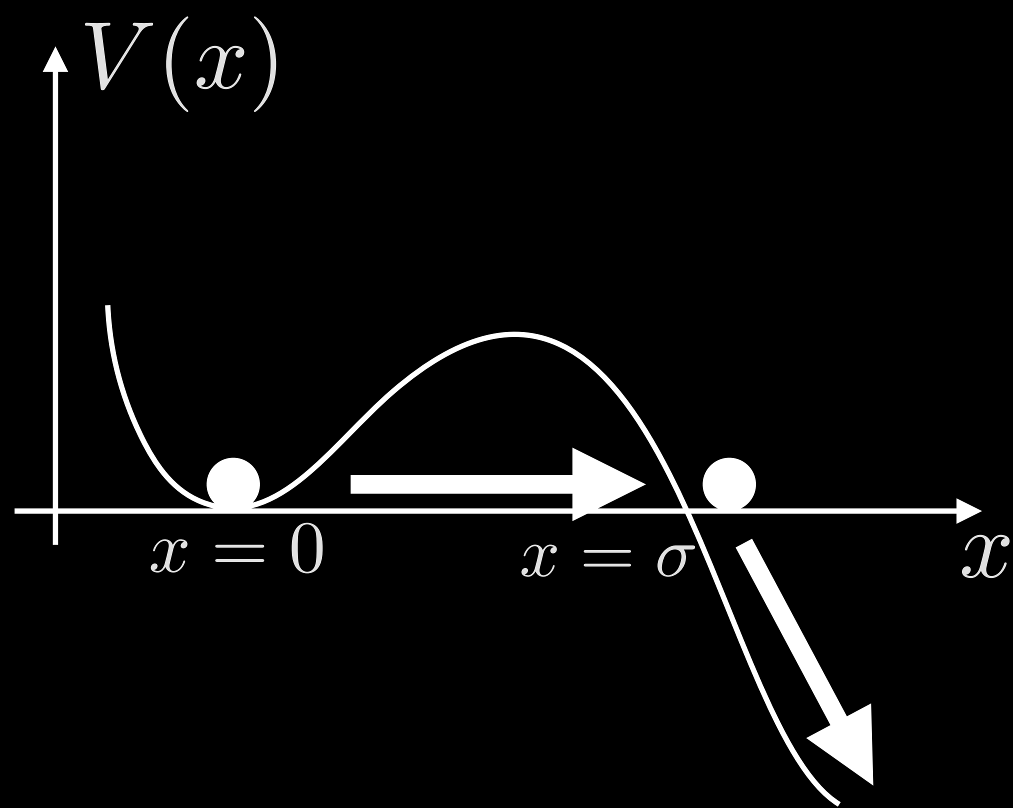
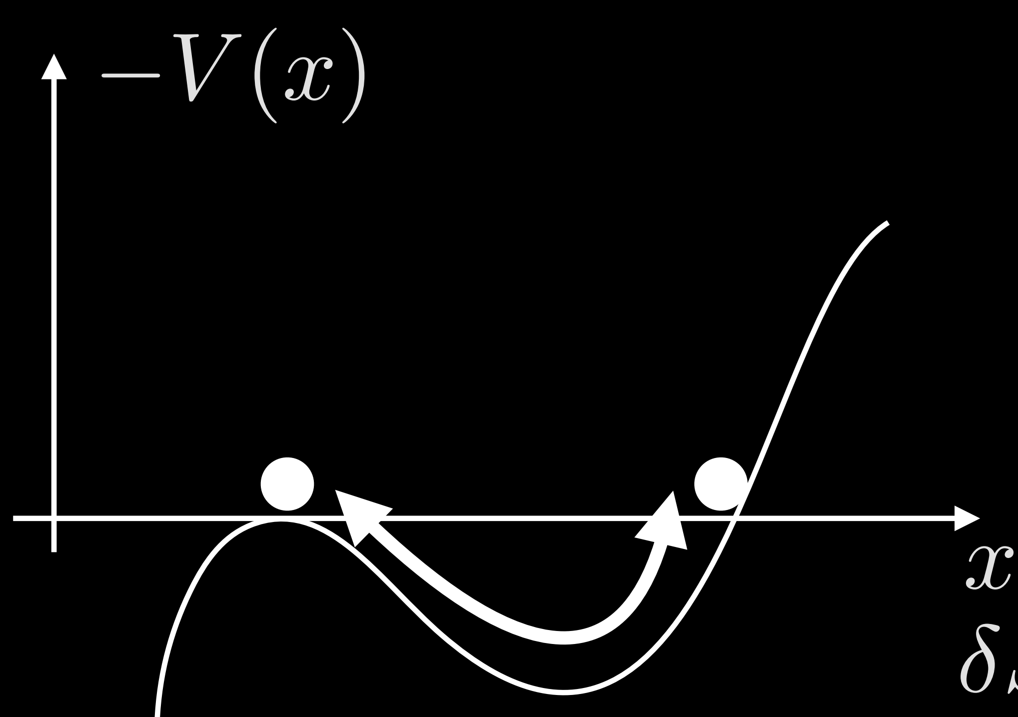
Quantum tunneling



$$\int e^{iS(x)} \mathcal{D}x$$

oscillatory

$$\Big\downarrow \quad t \to -i\tau$$



$$\int e^{-S_{\mathrm{E}}(x)} \mathcal{D}x \sim A e^{-\bar{S}_{\mathrm{E}}}$$ bounce

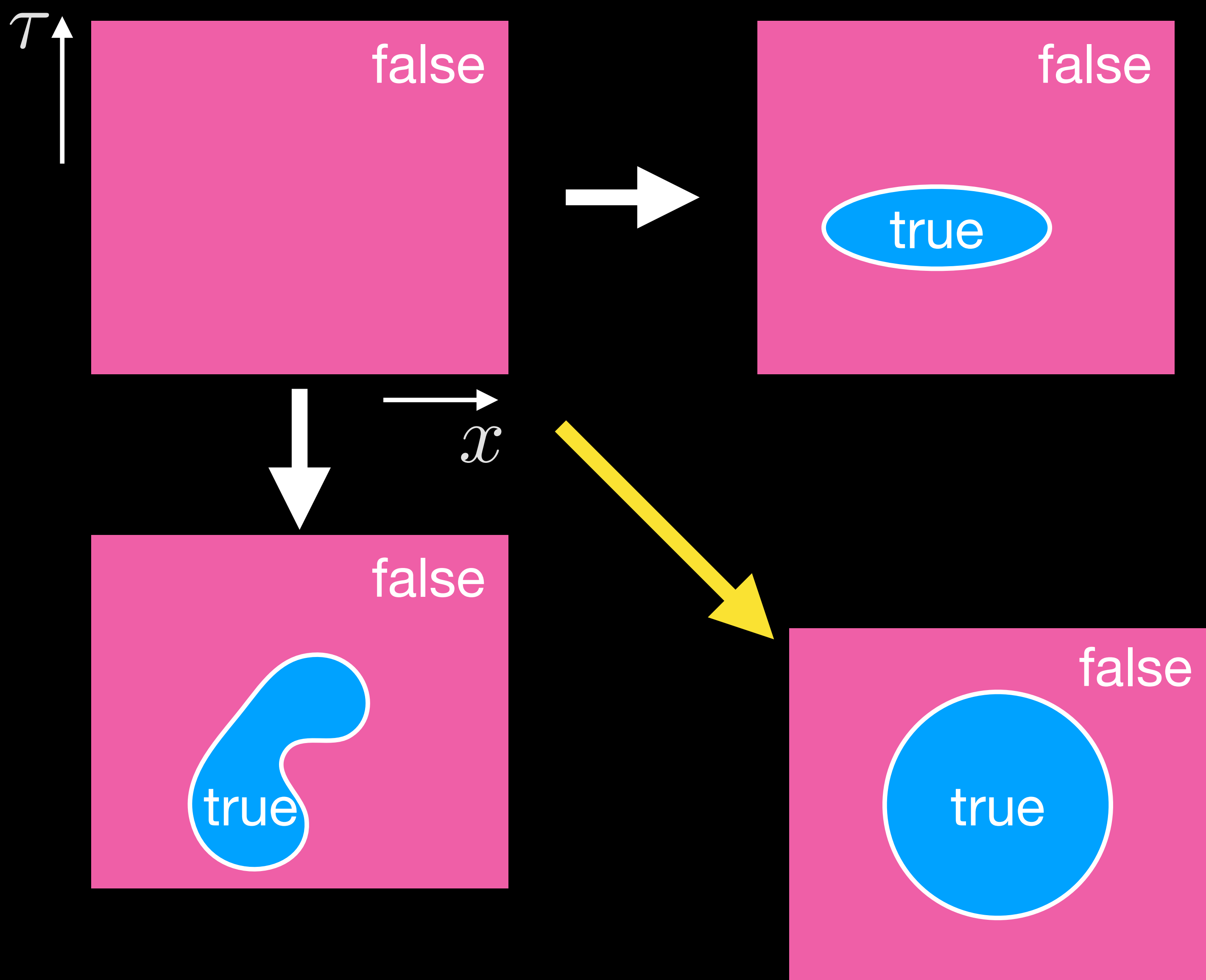
exponential

$$\bar{S}_{\mathrm{E}} = 2 \int_0^\sigma dx \sqrt{2V(x)}$$

Classical Sol. of

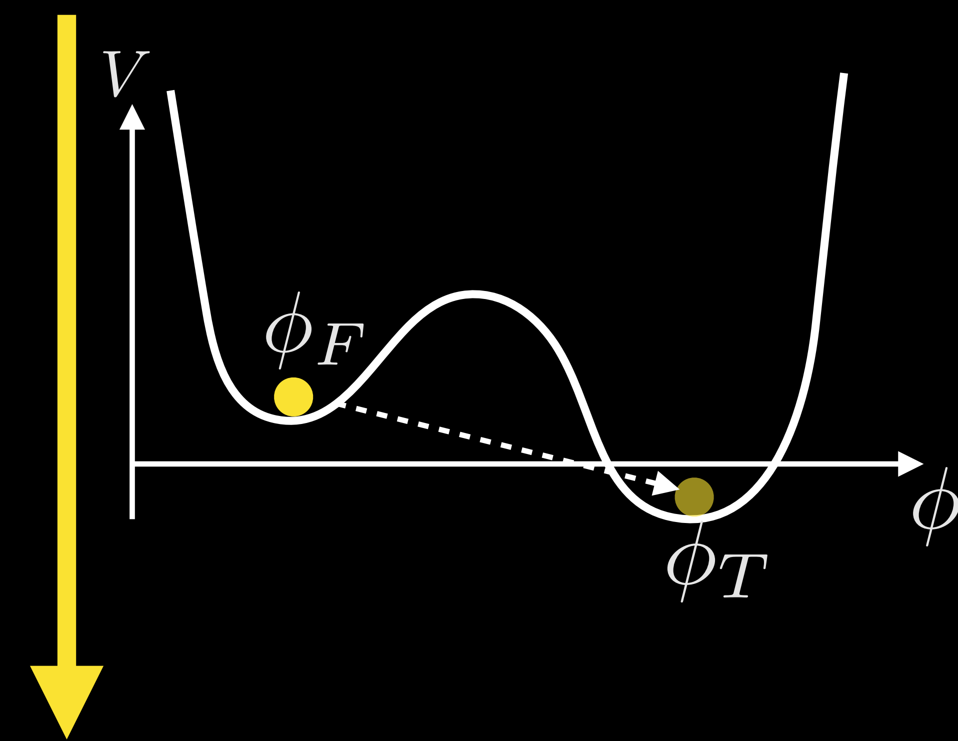
$$\frac{\delta S_{\mathrm{E}}}{\delta x} = 0 \rightarrow \left(\frac{dx}{d\tau}\right)^2 - V(x) = 0$$

Extension to field theory



Highest symmetry
O(4) in Euclidean space
Coleman, Glaser, Martin (1978)

$$\left(\frac{\partial^2}{\partial \tau^2} + \nabla^2\right)\phi = V'(\phi)$$



$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = V'(\phi)$$

friction potential force

$$\rho = \sqrt{\tau^2 + r^2}$$

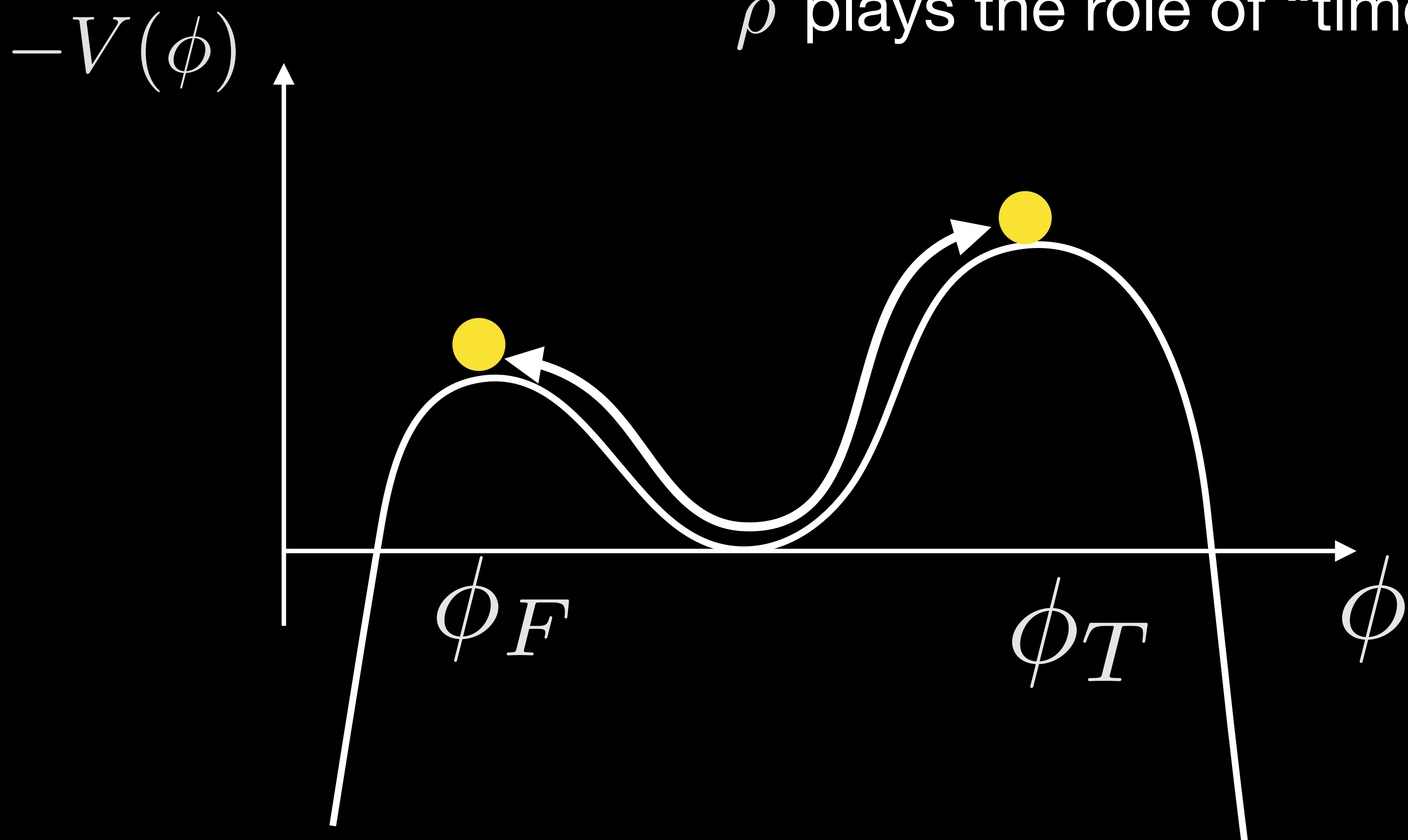
Construction of a bubble solution

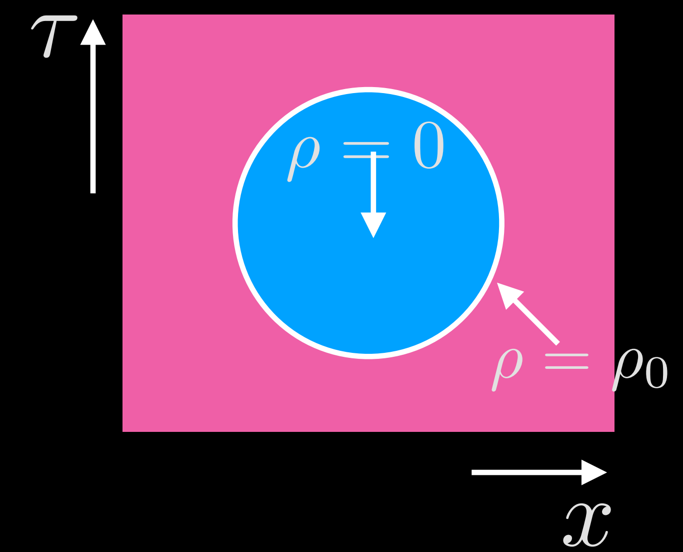
$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = V'(\phi)$$

friction

potential force

ρ plays the role of "time"





Boundary condition

$d\phi/d\rho \to 0 \quad \rho \to 0$

$\phi \to \phi_F \quad \rho \to \infty$

Vacuum decay with gravity (Coleman de Luccia solution)

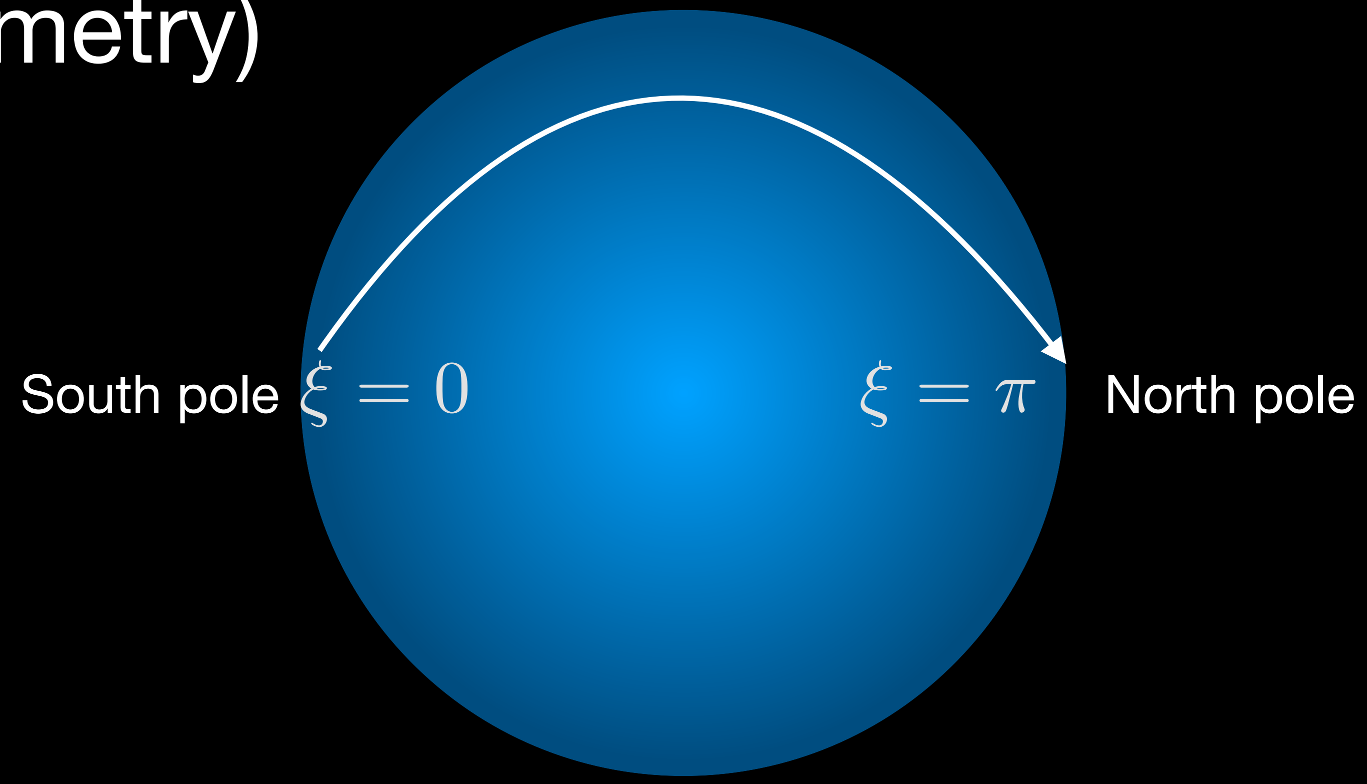
Case of dS → dS

line element on a unit sphere of S3

$$ds^2 = d\xi^2 + \rho^2(\xi) d\Omega_{III}^2$$

To be determined by solving the Einstein eq.

(Euclid) de Sitter (O(5) symmetry)



CdL solution

e.o.m. of the scalar field

$$\phi''(\xi) + \frac{3\rho'(\xi)}{\rho(\xi)}\phi'(\xi) = \frac{dV}{d\phi}$$

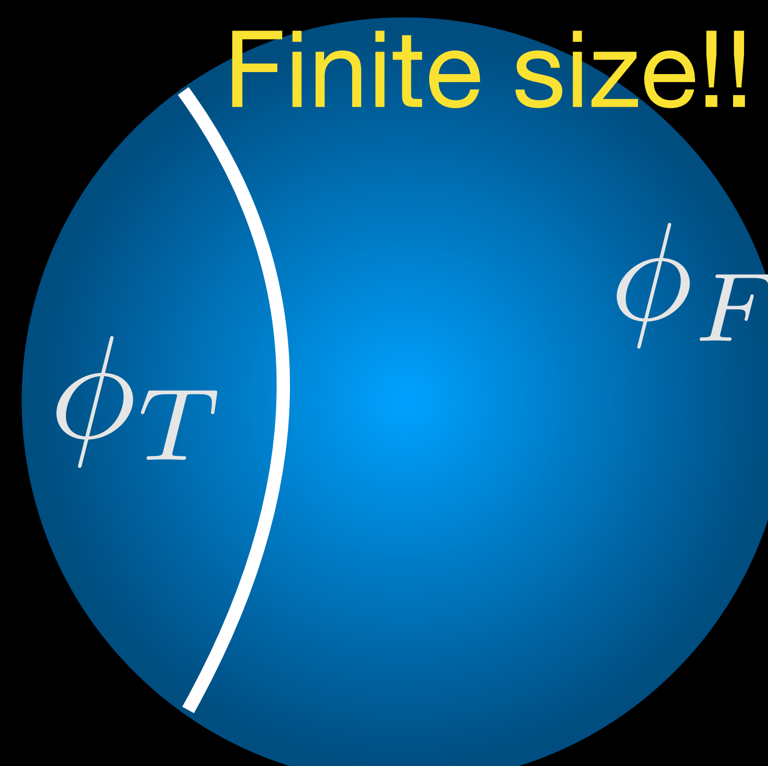
$\xi\xi$ -component of the Einstein equation

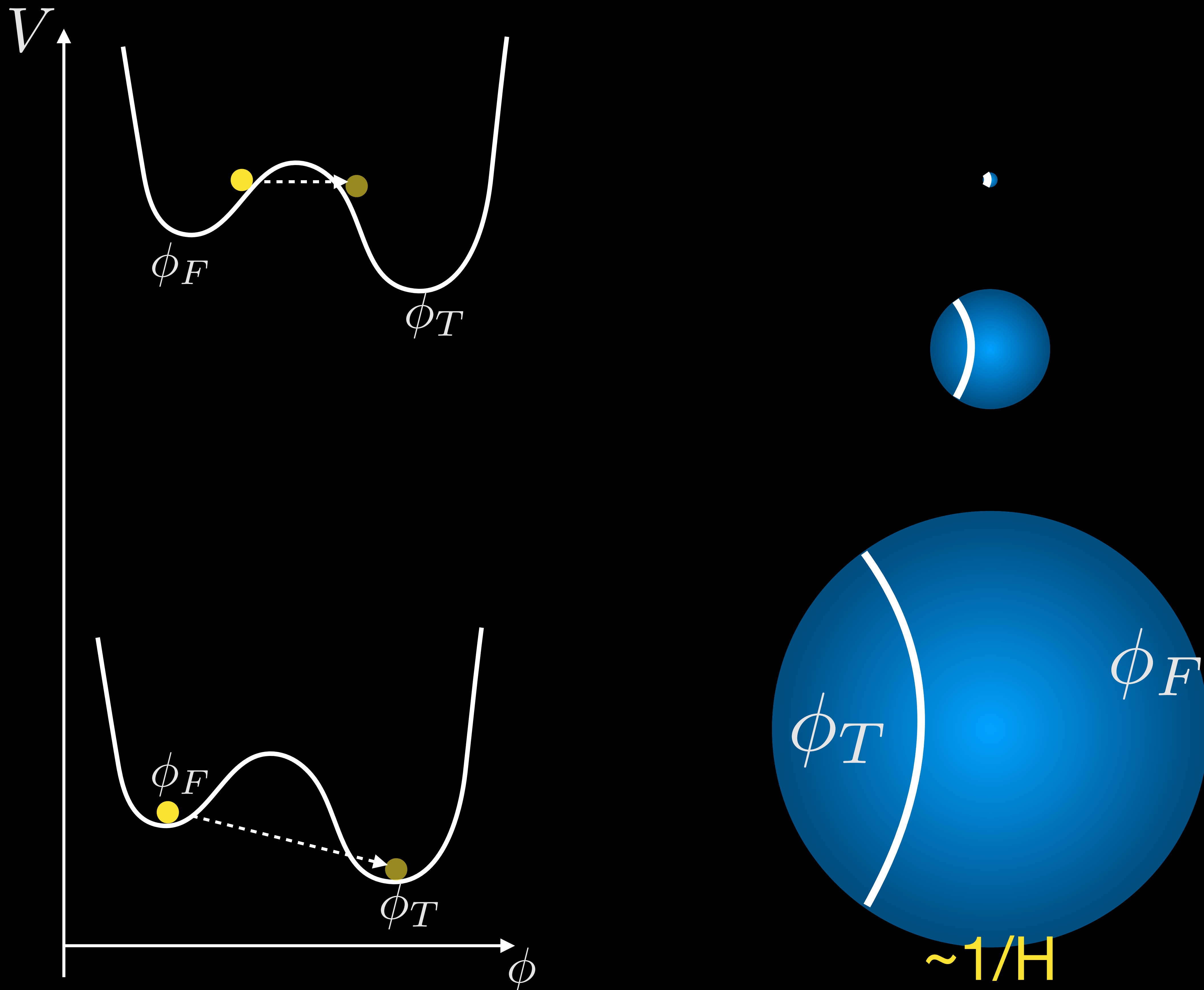
$$\rho'(\xi)^2 = 1 + \frac{8\pi G}{3}\rho^2\left(\frac{1}{2}\phi'^2 - V(\phi)\right)$$

$$H^2 = \frac{8\pi G}{3}V$$

Boundary condition

$$\phi' \to 0 \quad \rho \to 0$$



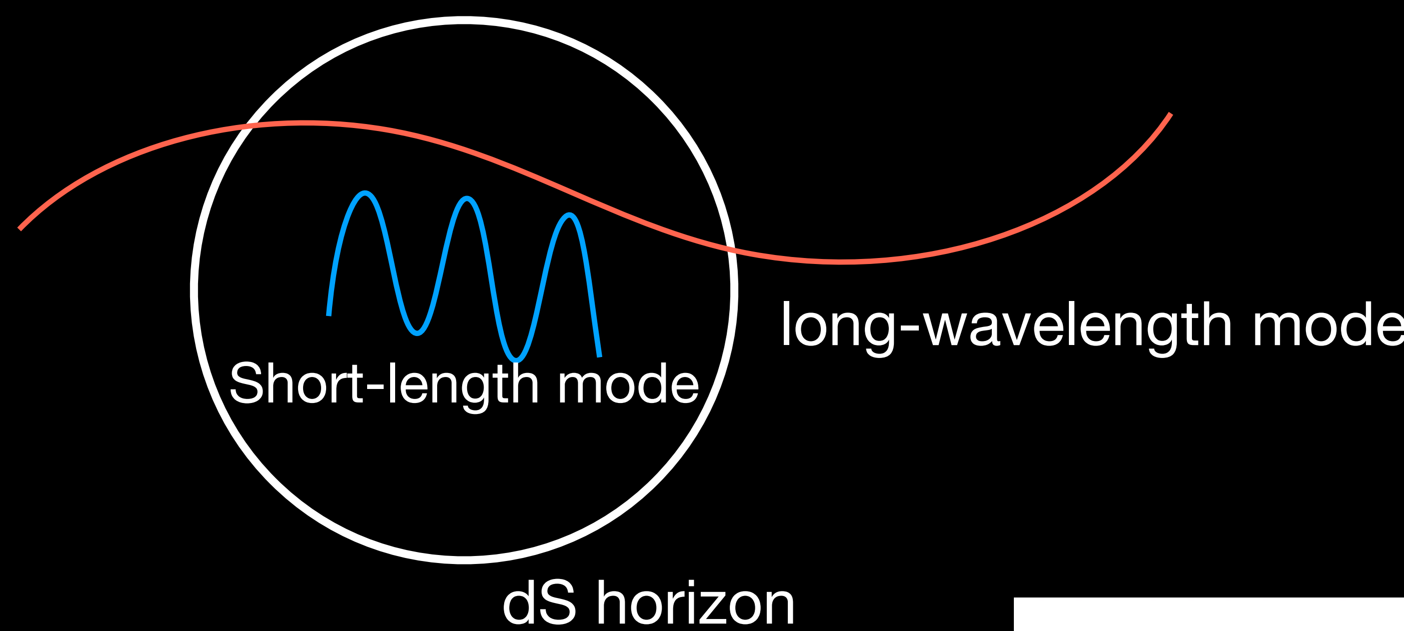


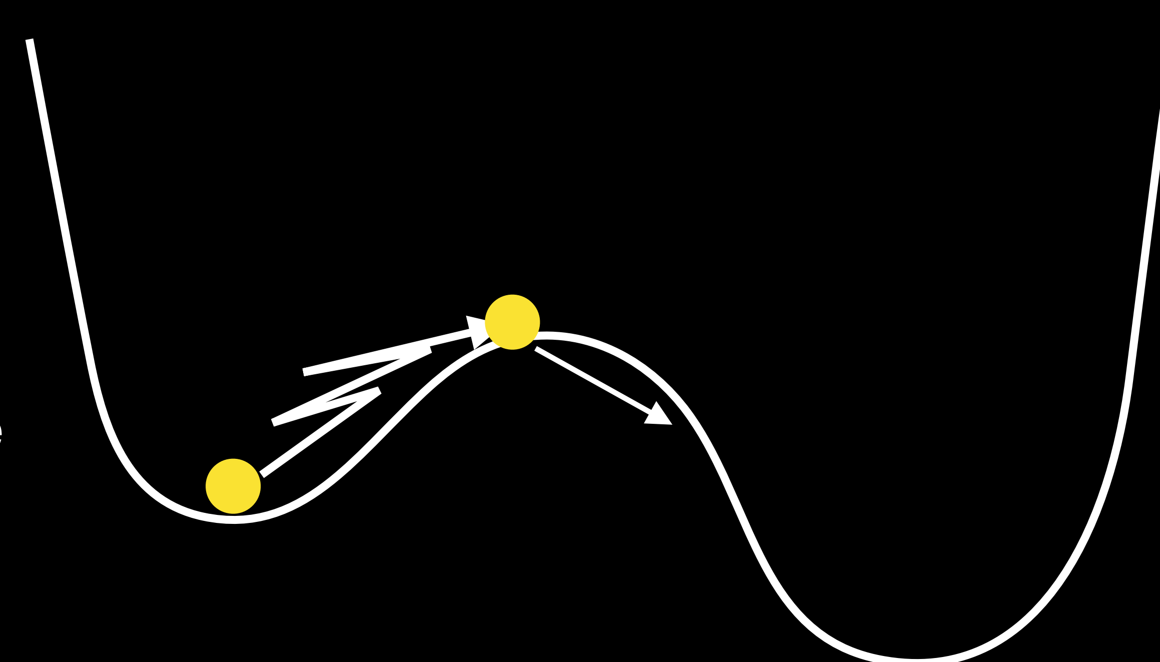
$$H^2 = \frac{8\pi G}{3} V$$

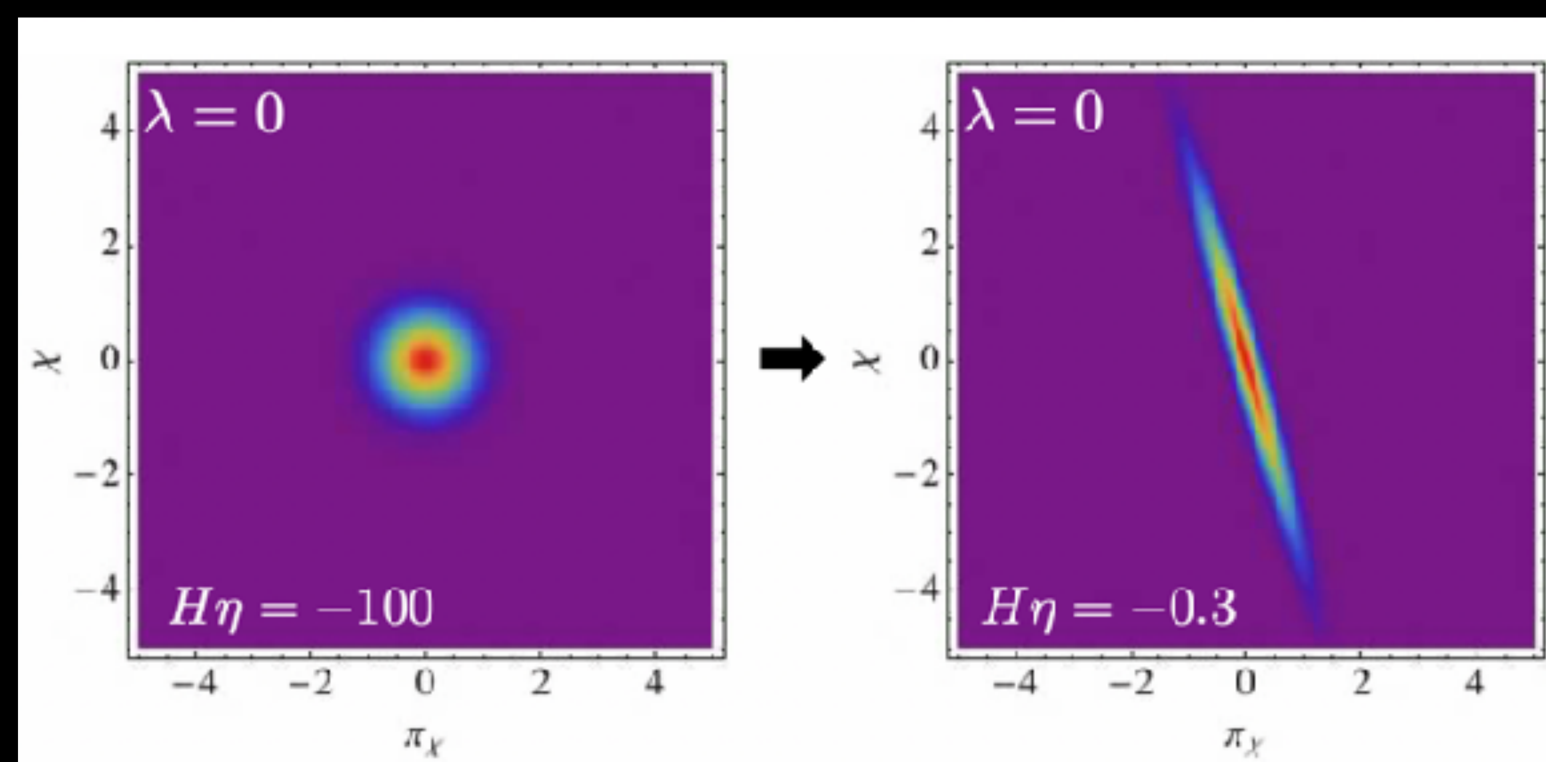
Stochasticity of quantum fluctuations on dS

c.f. Starobinsky & Yokoyama (1994)

Quantum fluctuations on dS background

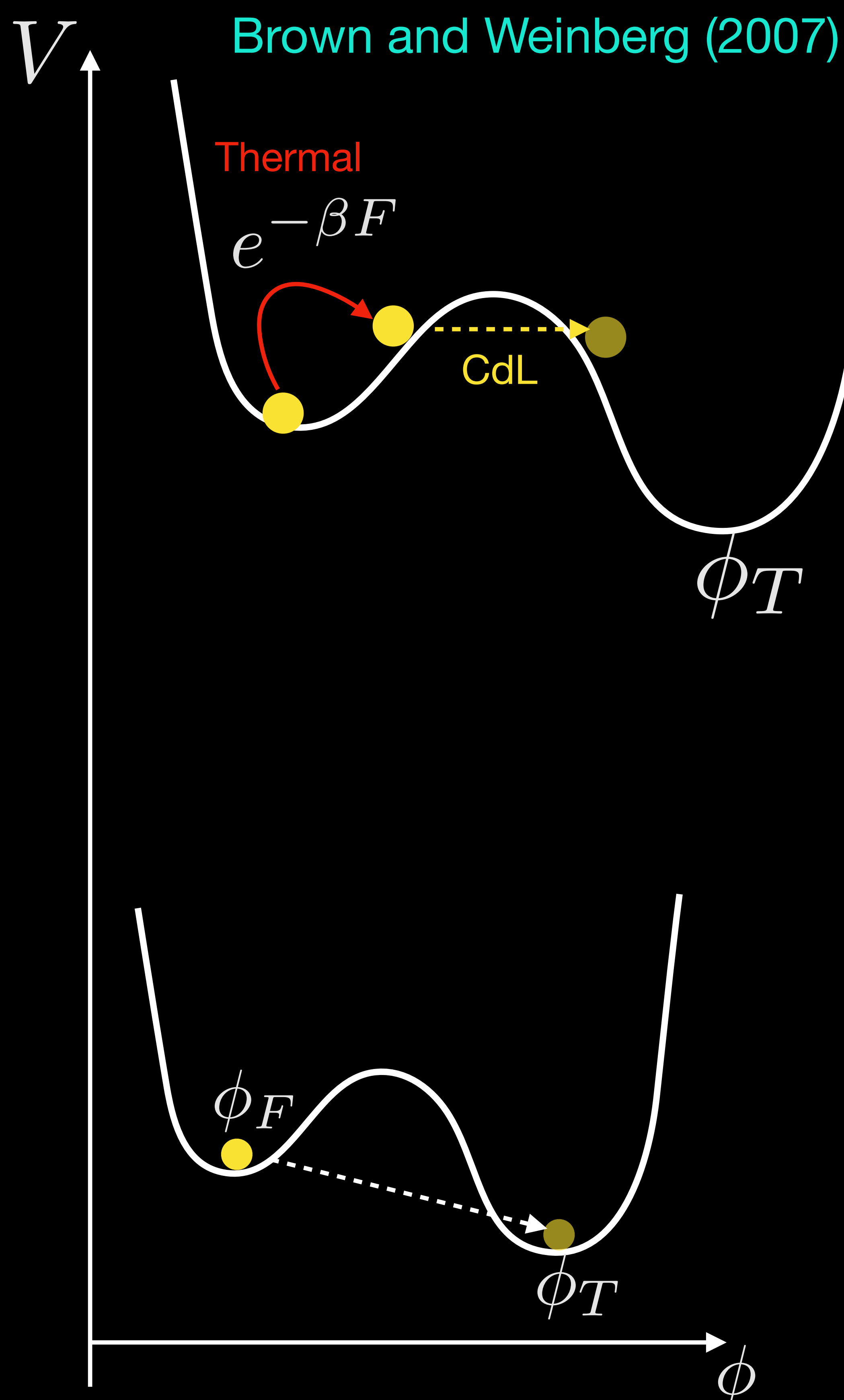


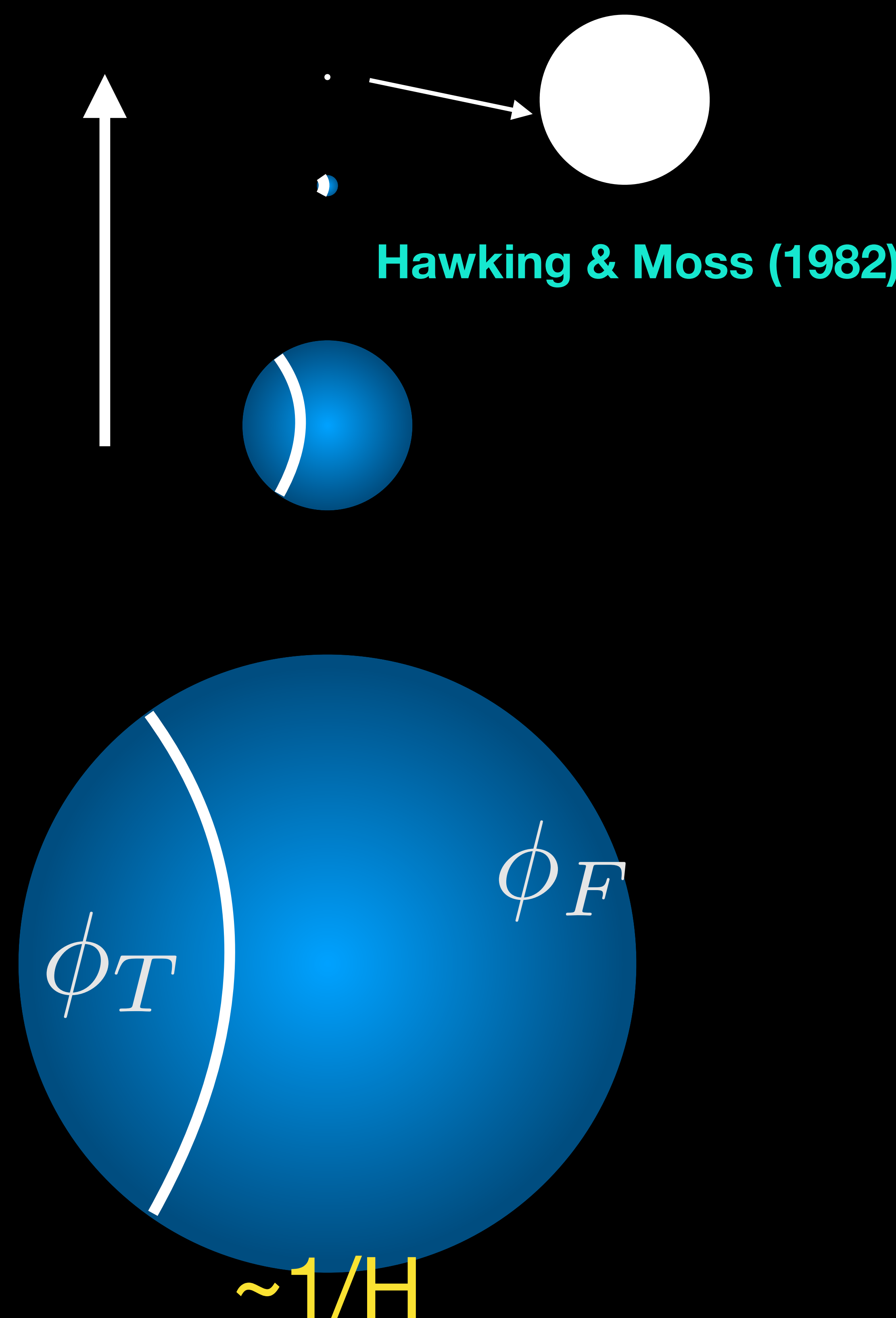




Wigner function of quantum fluctuations on dS background N.O. (2017)

Hawking-Moss solution

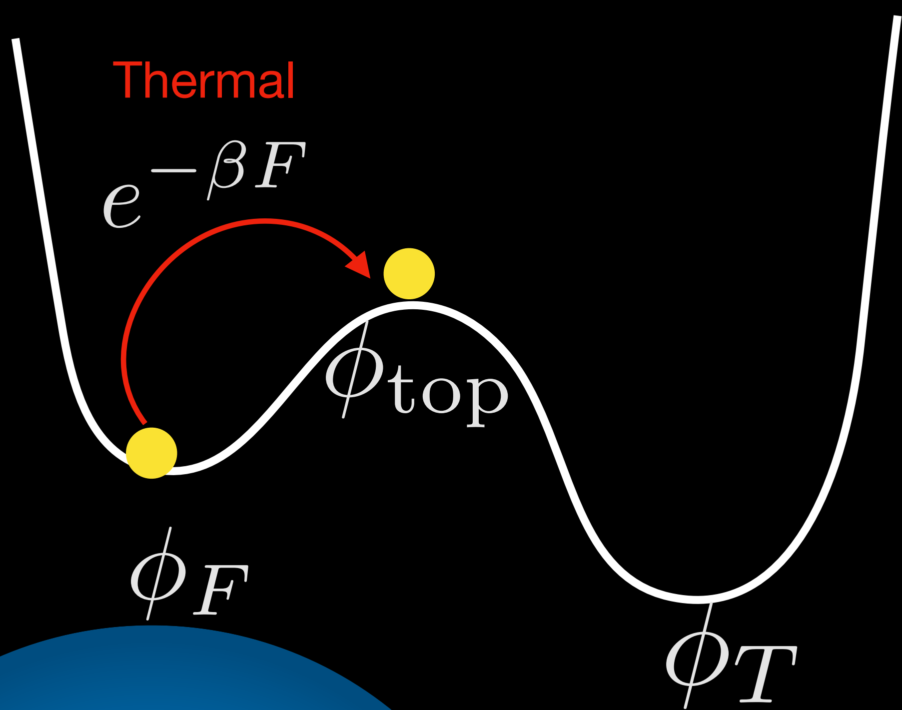




$$H^2 = \frac{8\pi G}{3} V$$

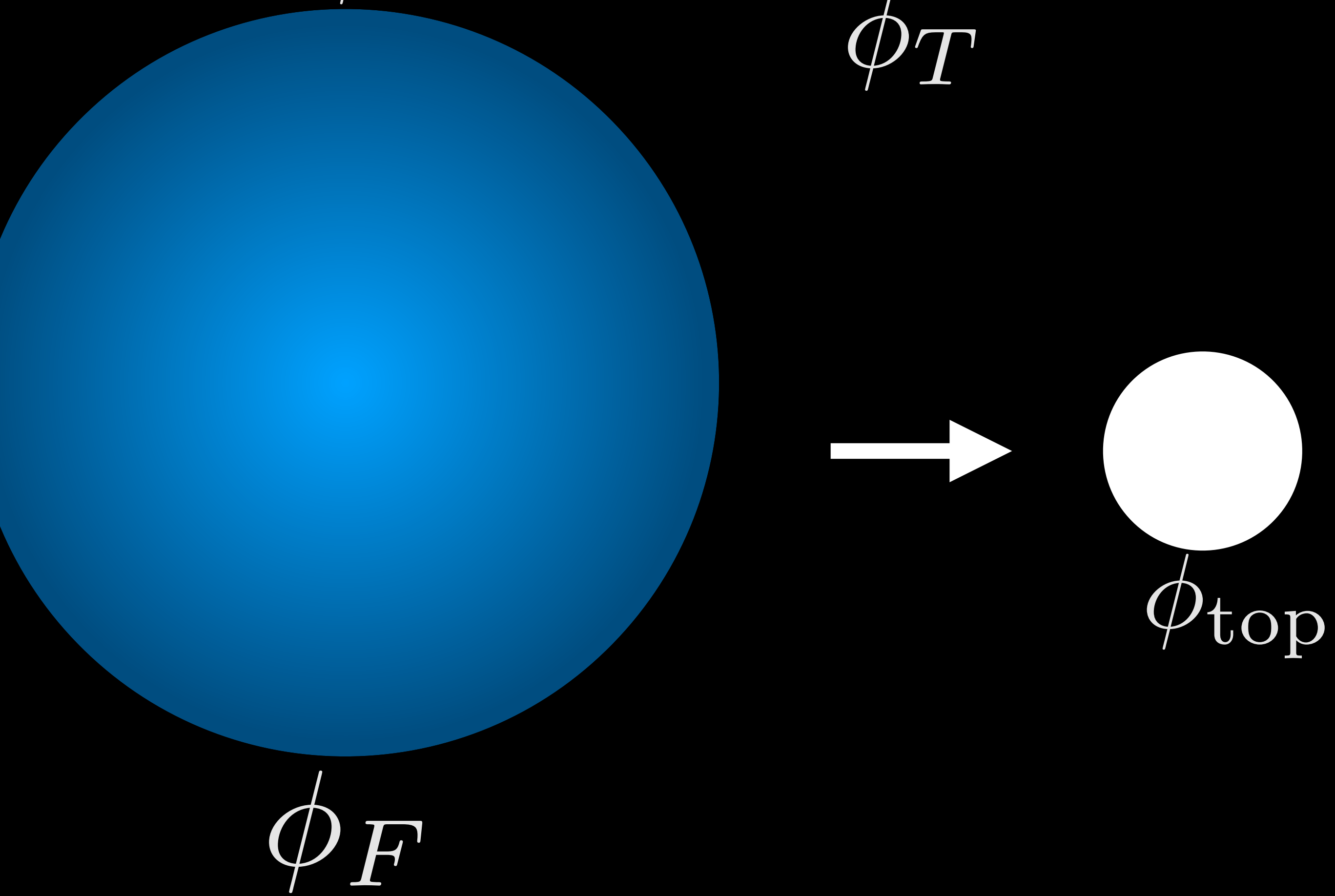
Hawking-Moss solution

Hawking & Moss (1982)



$$\beta F = S_{\rm E}[\phi_{\rm top}] - S_{\rm E}[\phi_{\rm F}]$$

$$S_{\rm E}[dS] = -\frac{A_{\rm cosmo}}{4G}$$

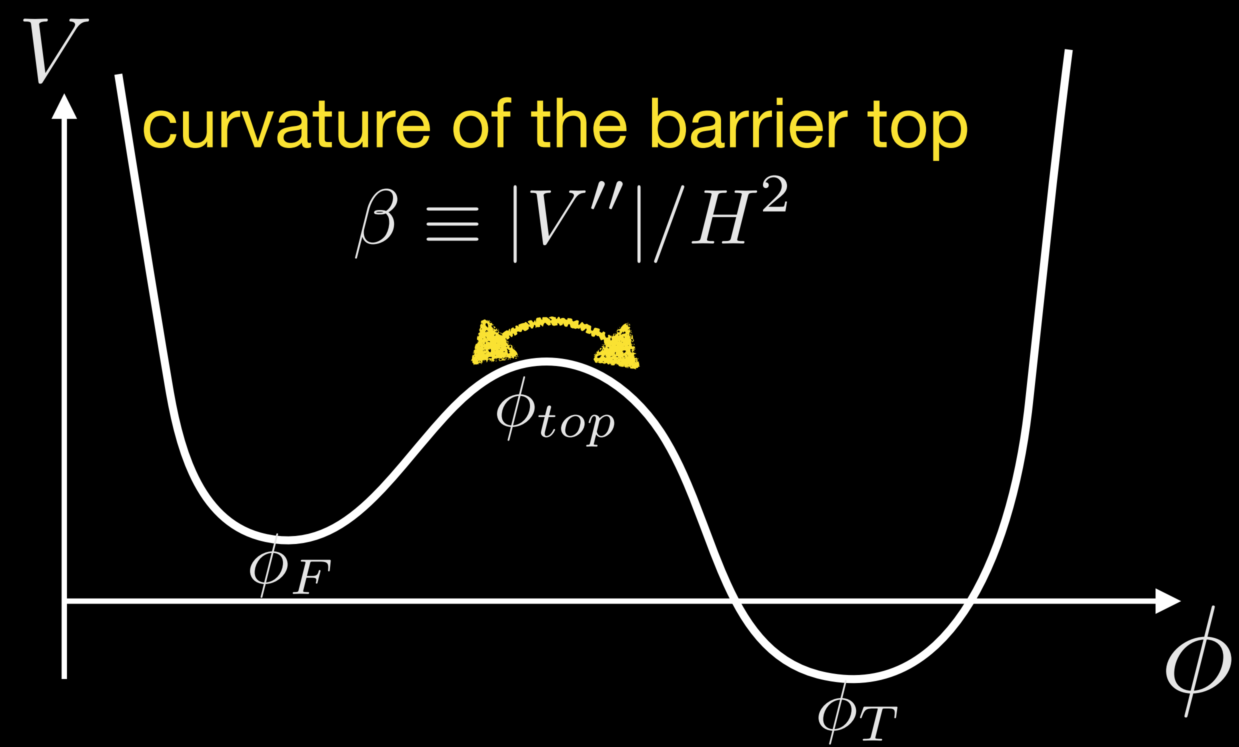


$$\beta F = -\frac{\Delta A_{\rm cosmo}}{4G} \simeq \frac{\Delta M}{T_{\rm dS}}$$

$\Delta H / H_{\rm top} \ll 1$

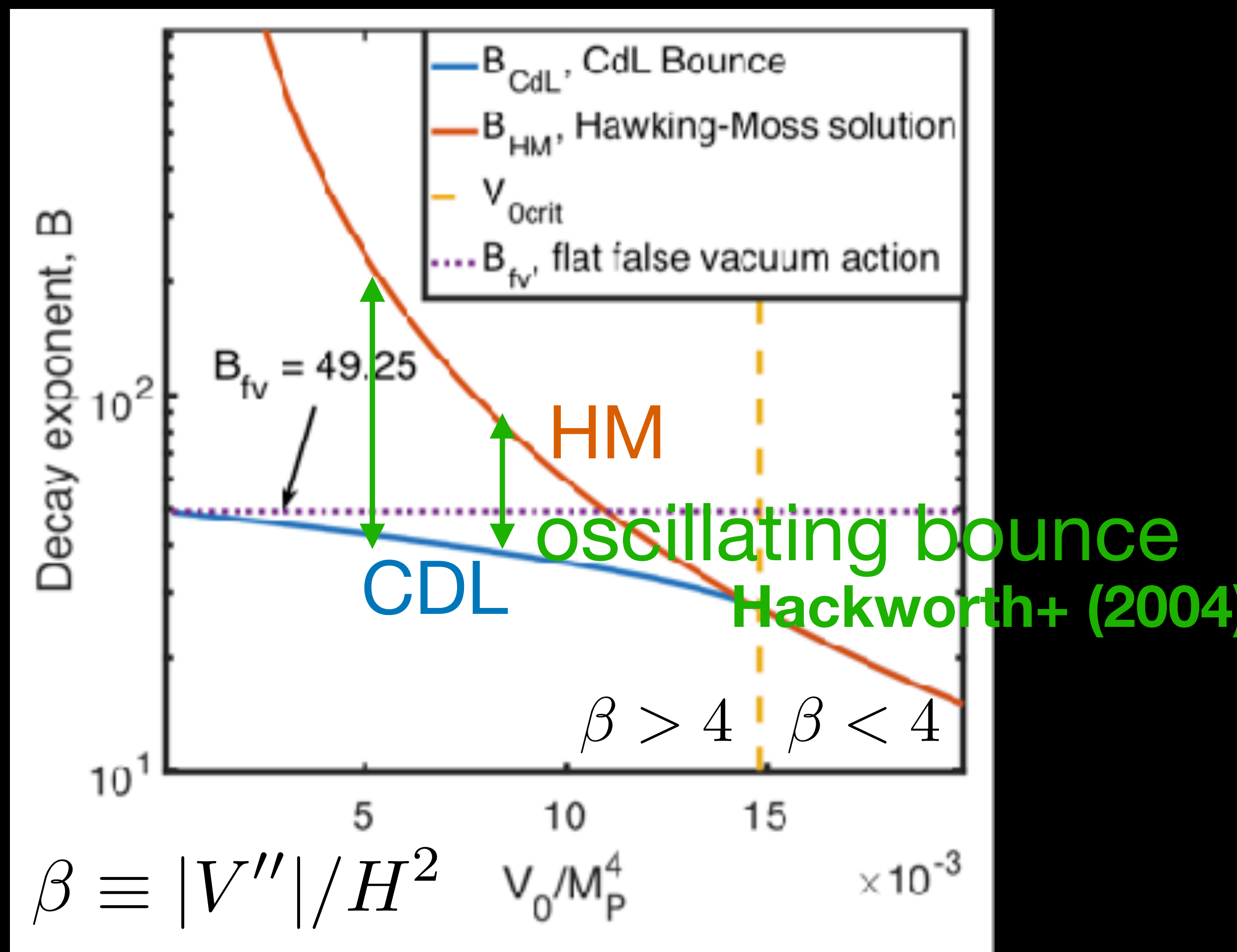
$$T_{\rm dS} = \frac{H}{2\pi}$$

Coleman-de Luccia vs Hawking-Moss



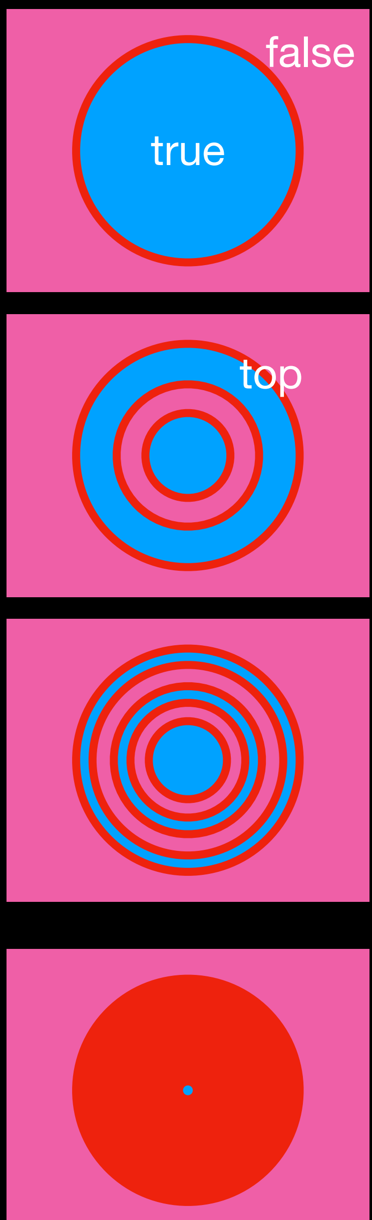
$\beta > 4 \cdots$ CDL and HM

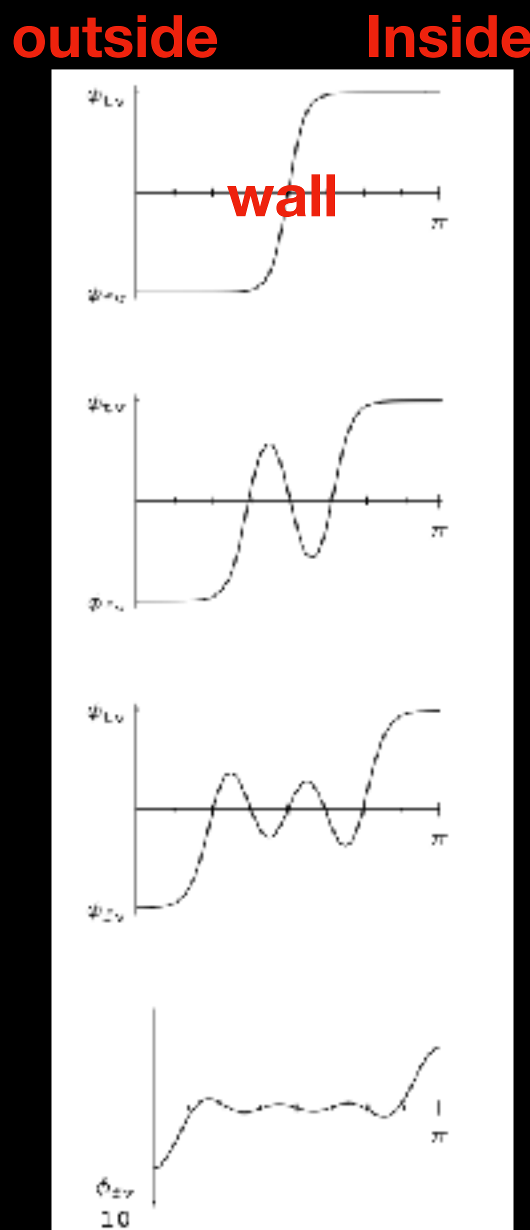
$\beta < 4 \cdots$ HM



A. Rajantie + (2017)

CDL, HM, and oscillating bounce





CDL

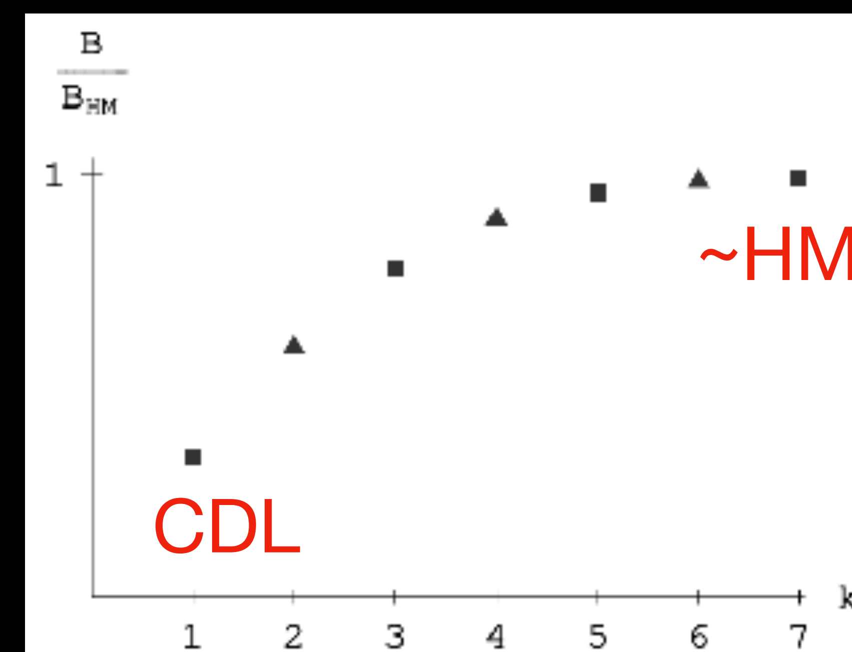
oscillating bounce (n=3)

n=5

n=7 ~ HM bounce

Hackworth+ (2004)

$$\Gamma \sim e^{-B}$$



oscillating bounces with many-oscillation limit → HM bounce

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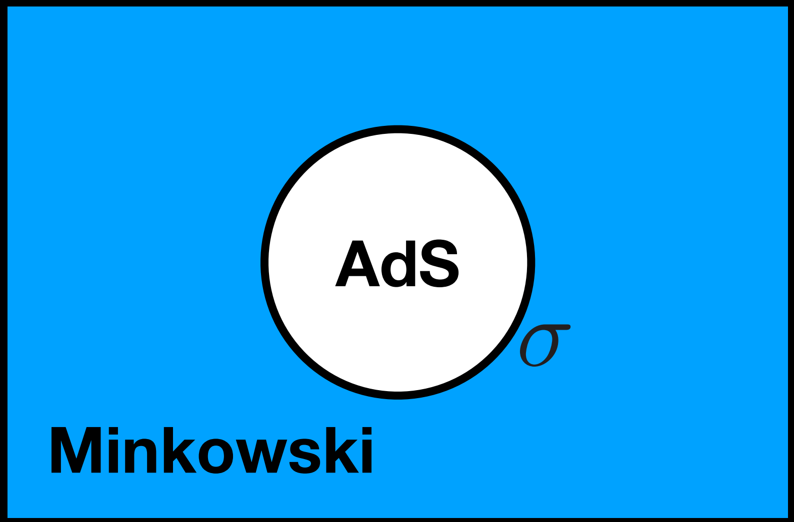
Computation of the CdL bounce in thin-wall limit

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$f(r) = \begin{cases} 1 & \cdots \text{ Minkowski} \\ 1 + H^2 r^2 & \cdots \text{ AdS} \end{cases}$$



Israel junction condition (Einstein equation)

$$\frac{\sqrt{1 + H^2 R^2 + \dot{R}^2}}{R} - \frac{\sqrt{1 + \dot{R}^2}}{R} = -4\pi G\sigma$$

↑ energy density of wall

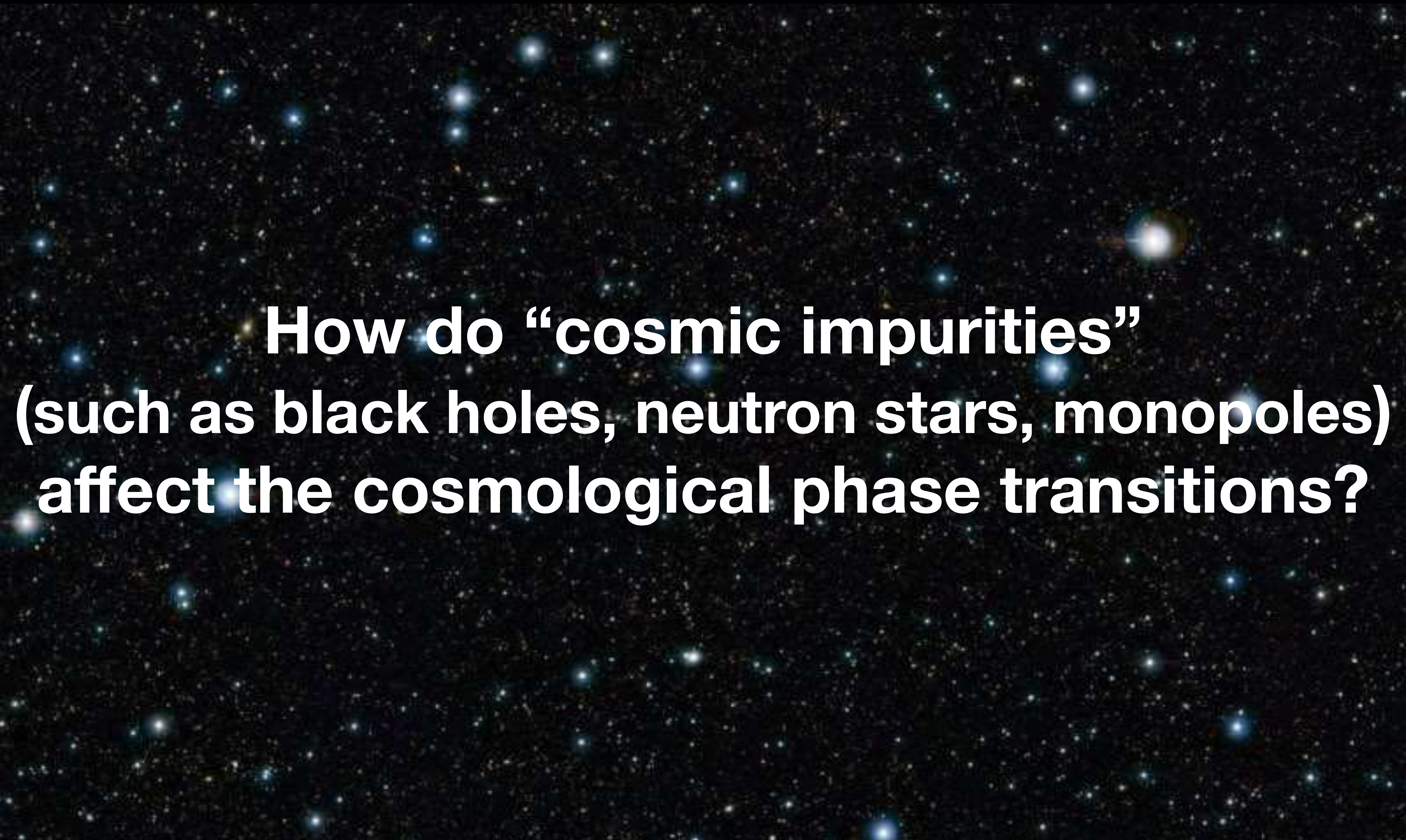
$$R = \gamma^{-1} \cosh \gamma\tau \quad \cdots \text{ Lorentzian}$$

$$R = \gamma^{-1} \cos \gamma\tau_{\mathrm{E}} \quad \cdots \text{ Euclidean}$$

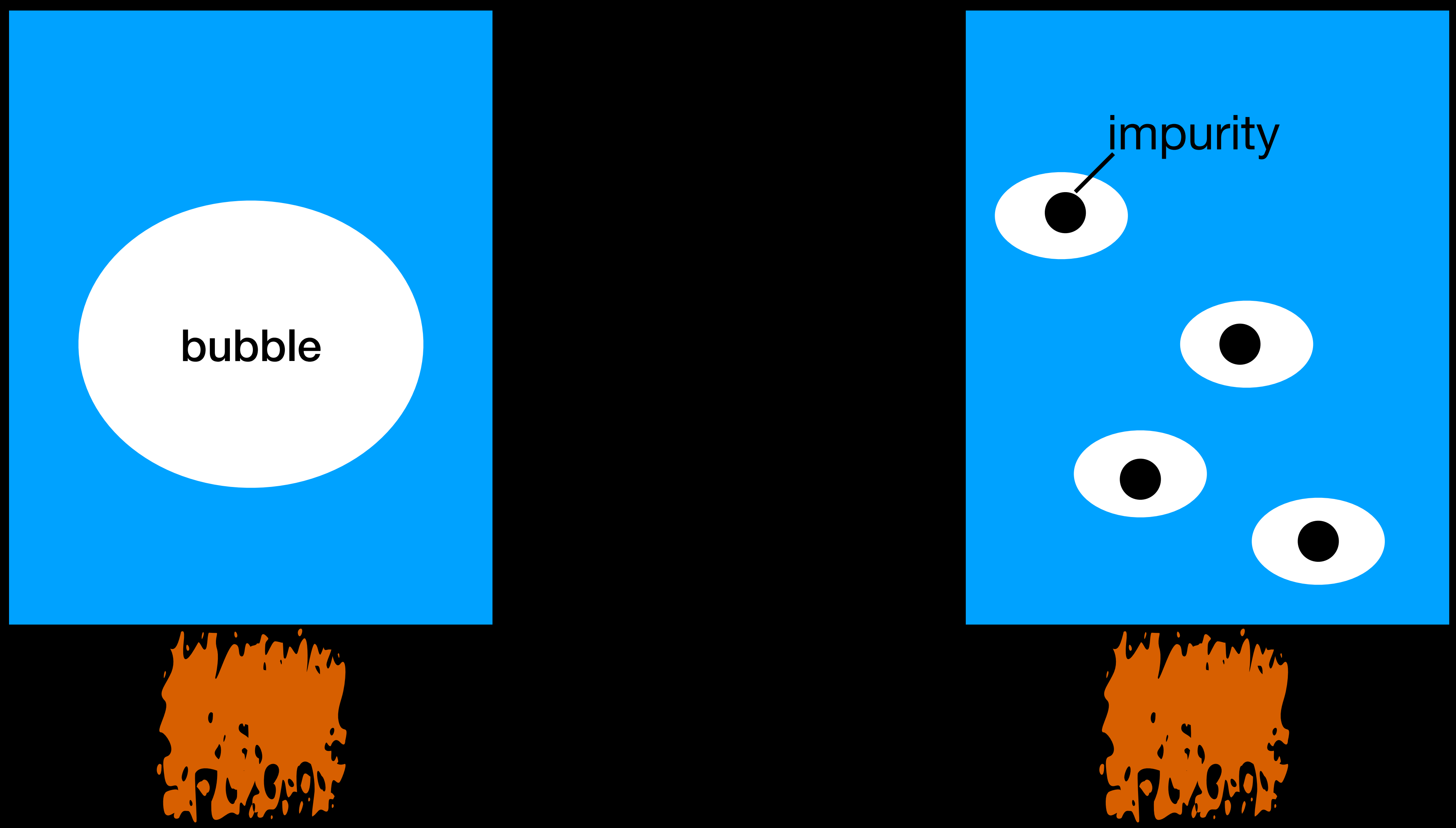
$$\gamma = \frac{|H^2 - (4\pi G\sigma)^2|}{8\pi G\sigma}$$

CDL action

$$S_{\mathrm{E}}[\phi_f] = \frac{\pi}{GH^2} \frac{(4\pi G\sigma)^4 / H^4}{(1 - (4\pi G\sigma)^2 / H^2)^2}$$


How do “cosmic impurities”
(such as black holes, neutron stars, monopoles)
affect the cosmological phase transitions?

Catalysis



small bubbles are easy to nucleate!!

Black holes as bubble nucleation cites

Hiscock (1987) & Gregory et al. (2014)

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$f(r) = \begin{cases} 1 - 2GM_+/r & \cdots \text{ Schwarzschild} \\ 1 - 2GM_-/r + H^2 r^2 & \cdots \text{ AdS-Schwarzschild} \end{cases}$$

Junction condition

$$\dot{R}^2 + V(R) = 0,$$

$$V(R) \equiv f_- - \frac{1}{4\Sigma^2 R^2}\left(\frac{2G(M_+ - M_-)}{R} + (H_-^2 - H_+^2 + \Sigma^2)R^2\right)^2,$$

$$\Sigma \equiv 4\pi G\sigma$$

$$\dot{R}^2 \pm V(R) = 0$$

Lorentzian ($+$)

Euclidean ($-$)

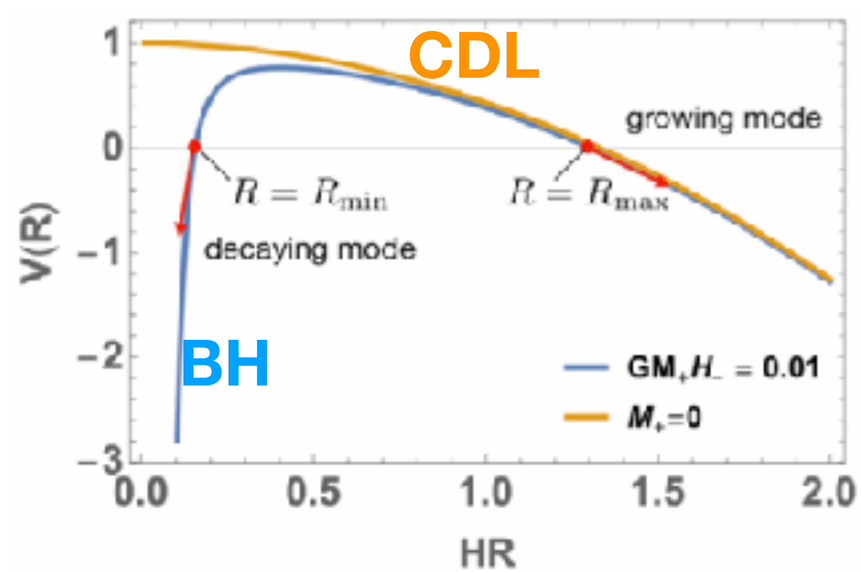


Figure 1. Plot of the effective potential $V(R)$ with $H_+ = 0$, $M_- = 0$, $H_- = H > 0$, $\Sigma = H/2$, and $M_+ = 0.01/(GH)$.

N.O., K. Ueda, M. Yamaguchi (2019)

Thermal interpretation of vacuum decay

$$\Box\varphi = \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \Delta\varphi = \frac{\mathrm{d}V(\varphi,0)}{\mathrm{d}\varphi} \equiv V'(\varphi,0)$$

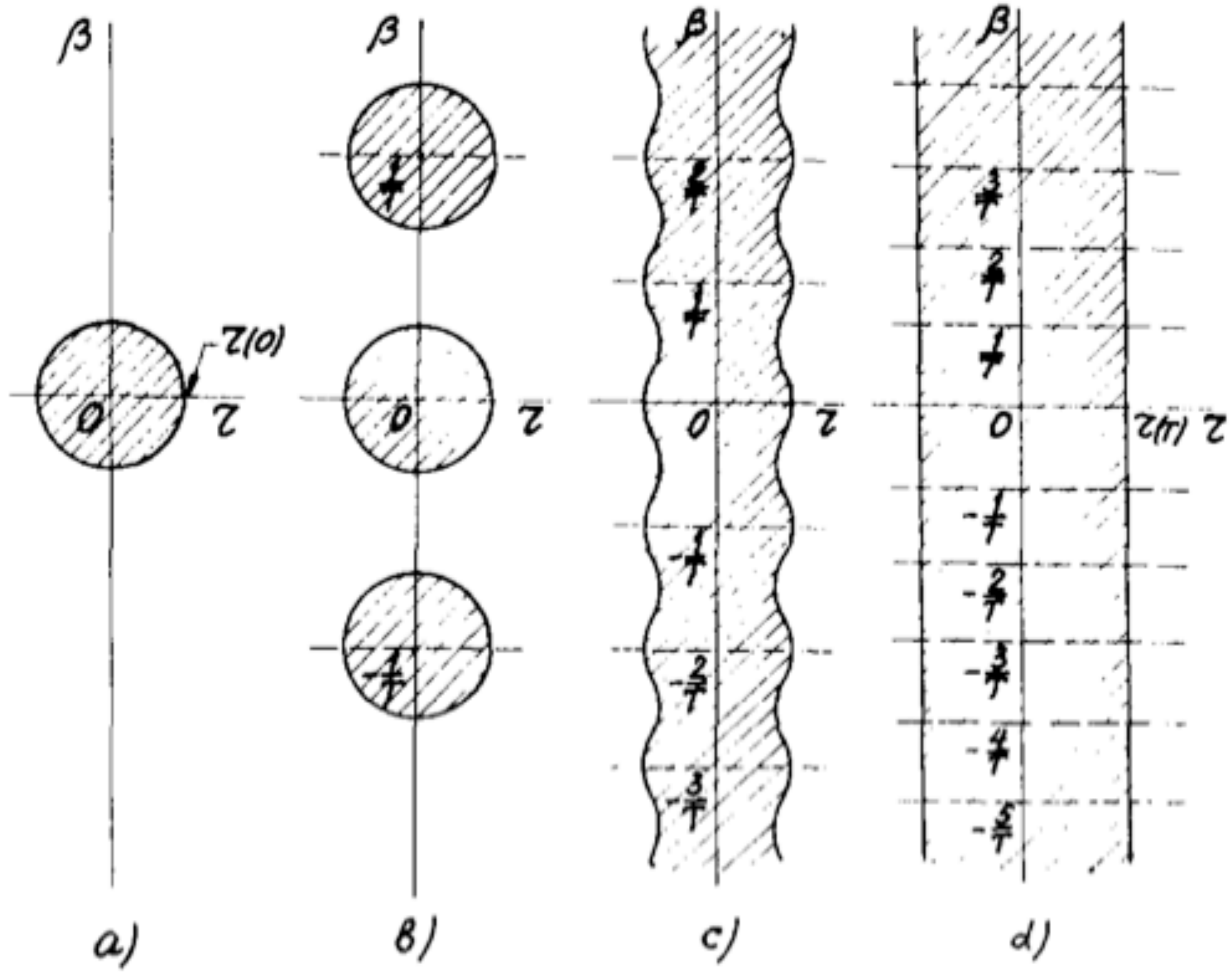


Fig. 3. Solution of (2.2) at different values of temperature. (a) $T=0$; (b) $T \ll r^{-1}(0)$; (c) $T \sim r^{-1}(0)$; (d) $T \gg r^{-1}(0)$. The dashed regions contain the classical field $\varphi \neq 0$. For simplicity we have shown the bubbles for the case, when their wall thickness is less than the bubble radius.

A. Linde, “Decay of the false vacuum at finite temperature” (1983)

Thermal interpretation of BH catalysts

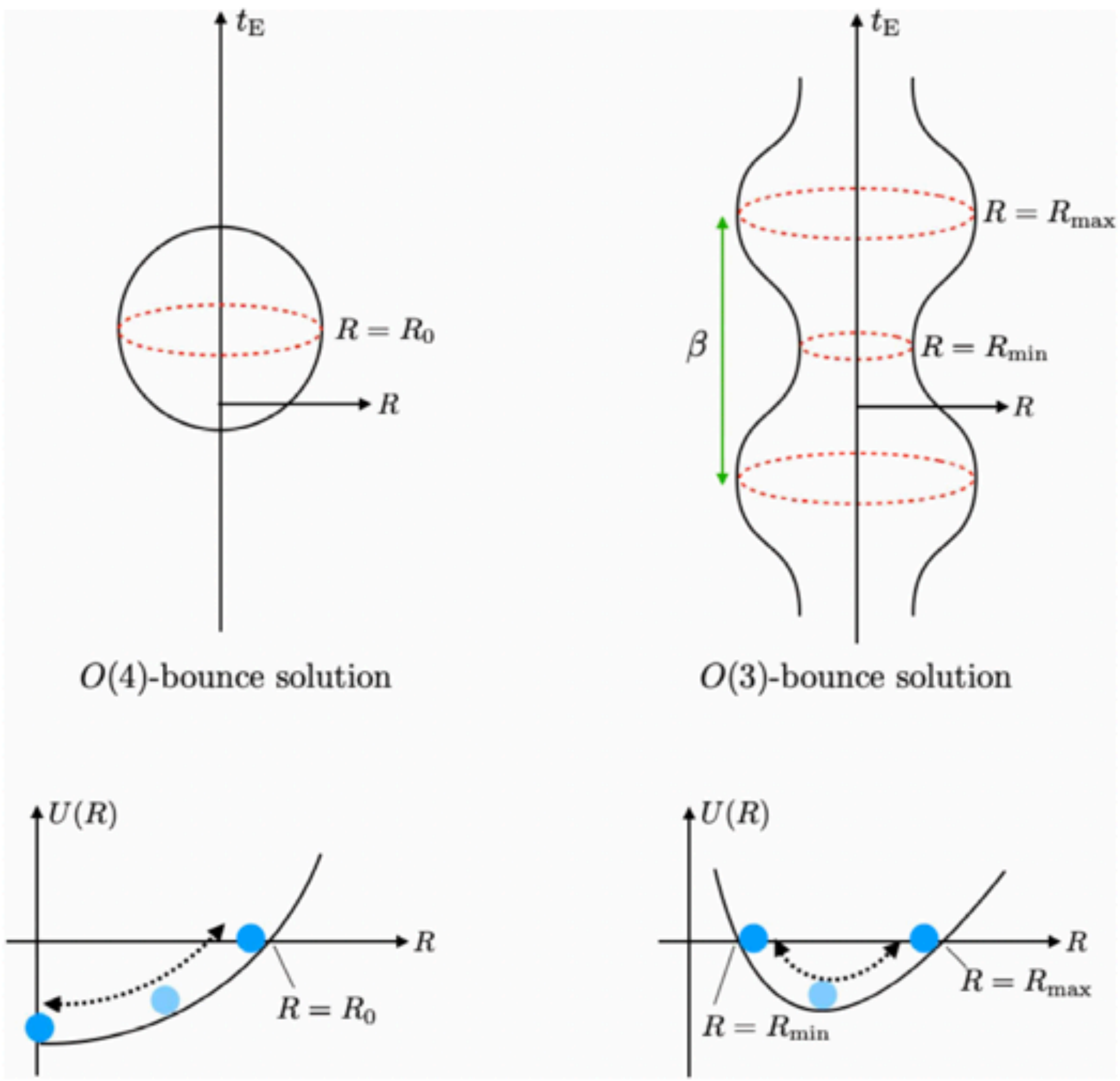


Figure 2. Schematic pictures showing the $O(4)$ and $O(3)$-bounce solutions.

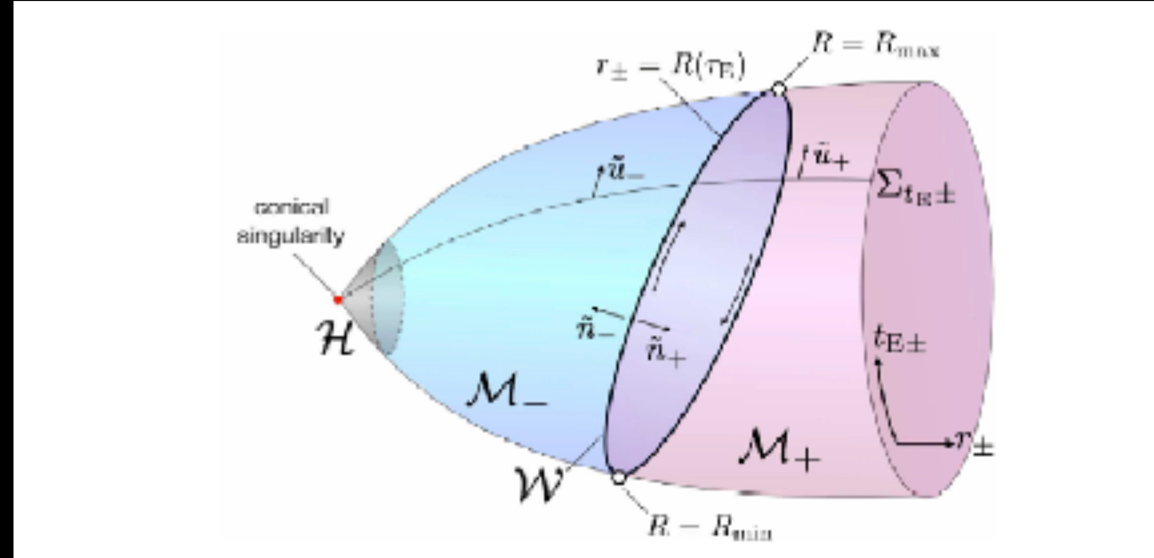


Figure 4. Schematic picture showing the decomposition of the $O(3)$-Euclidean manifold in the existence of the thin-wall vacuum bubble and the BH horizon.

N.O., K. Ueda, M. Yamaguchi (2019)

Euclidean action

Gregory, Moss, & Withers (2013)

$$I = -\frac{1}{4G}(\mathcal{A}_h + \mathcal{A}_c) - \frac{1}{2}\int_{\mathcal{W}} \sigma - \frac{1}{16\pi G}\int_{\mathcal{W}} (f'_+ \dot{\tau}_+ - f'_- \dot{\tau}_-)$$

matter

geometry

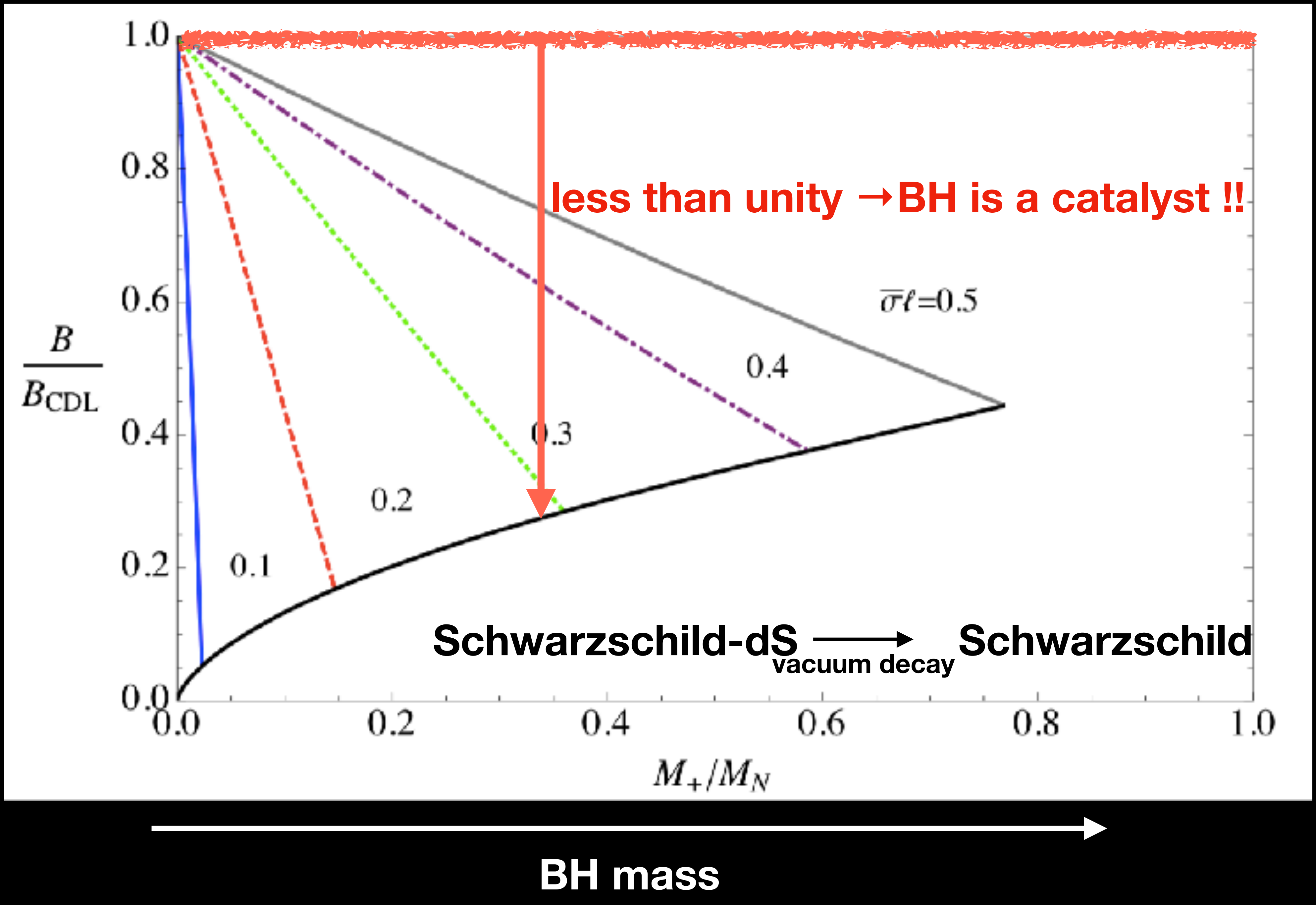
Gravitational horizons

bubble contribution

$$I = \frac{F}{T} = -S + \frac{E}{T}$$

$$\Gamma \sim e^{-B}$$

Gregory, Moss, & Withers (2013)



BH mass

Homogeneous background

Vacuum decay without gravity

Coleman (1977)

Vacuum decay/excitation with gravity

- Coleman de Luccia bounce
  Coleman & de Luccia (1980)
- Hawking-Moss bounce
  Hawking & Moss (1982)
- Oscillating bounce
  Hackworth & Weinberg (2004)

Inhomogeneous background

Vacuum decay catalyzed by a black hole

Hiscock (1987)

Gregory, Moss, & Withers (2013)

NO, Ueda, Yamaguchi (2019)

HM transition with a black hole

Gregory, Moss, NO (2020)

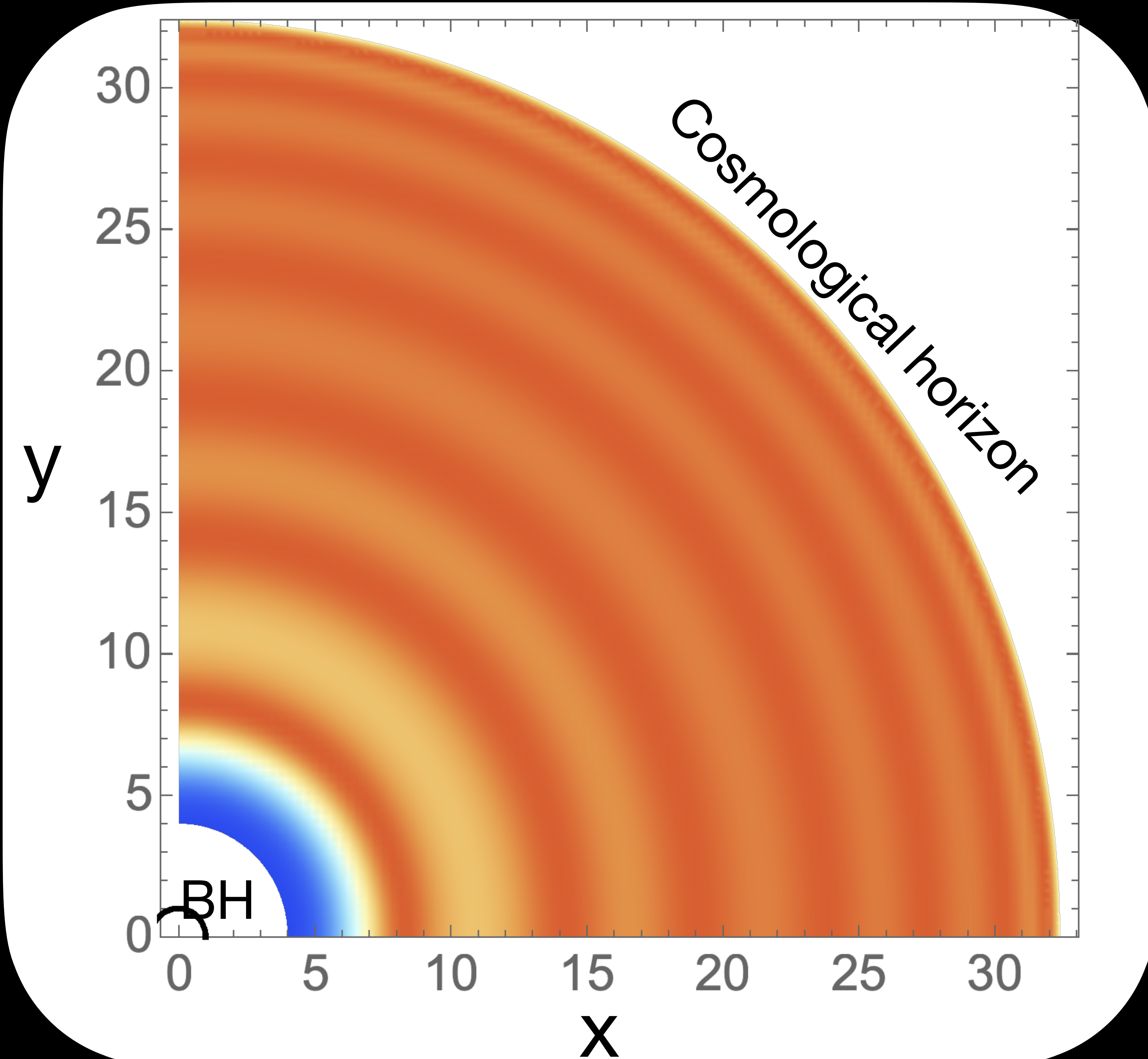
Gregory, Moss, NO, & Patrick (2020)

Oscillating bounce with a black hole

Gregory, Moss, NO (2020)

Applications:

PBH cosmology, Higgs metastability, initial condition problem for inflation, etc…



Cosmological horizon
BH
y
x
0
5
10
15
20
25
30

Static oscillating bounce in Schwarzschild-de Sitter space

static and spherical metric

$$ds^2 = f(r)e^{2\delta(r)}d\tau^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$f = 1 - \frac{2G\mu(r)}{r}$$

E.O.M. of a scalar field and Einstein equations

$$f\phi'' + f'\phi' + \frac{2}{r}f\phi' + \delta' f\phi' - V_\phi = 0$$

$$\mu' = 4\pi r^2\left(\frac{1}{2}f\phi'^2 + V\right)$$

$$\delta' = 4\pi G r\phi'^2$$

tortoise coordinagte

$$dr^* = dr/f(r)$$

Potential is shallow at horizons

$$\frac{d^2\phi}{dr^{*2}} + f(r)\frac{2}{r}\frac{d\phi}{dr^*} + 4\pi G f^{-1}(r)r\left(\frac{d\phi}{dr^*}\right)^3 - f(r)V_\phi(\phi) = 0$$

$$\frac{d\mu}{dr^*} = 4\pi r^2\left(\frac{1}{2}\left(\frac{d\phi}{dr^*}\right)^2 + f(r)V(\phi)\right)$$

$$V(\phi) = \beta H^2 \left(-\frac{1}{2}\phi^2 - \frac{g}{3v}\phi^3 + \frac{1}{4v^2}\phi^4 \right) + V_{\rm top},$$

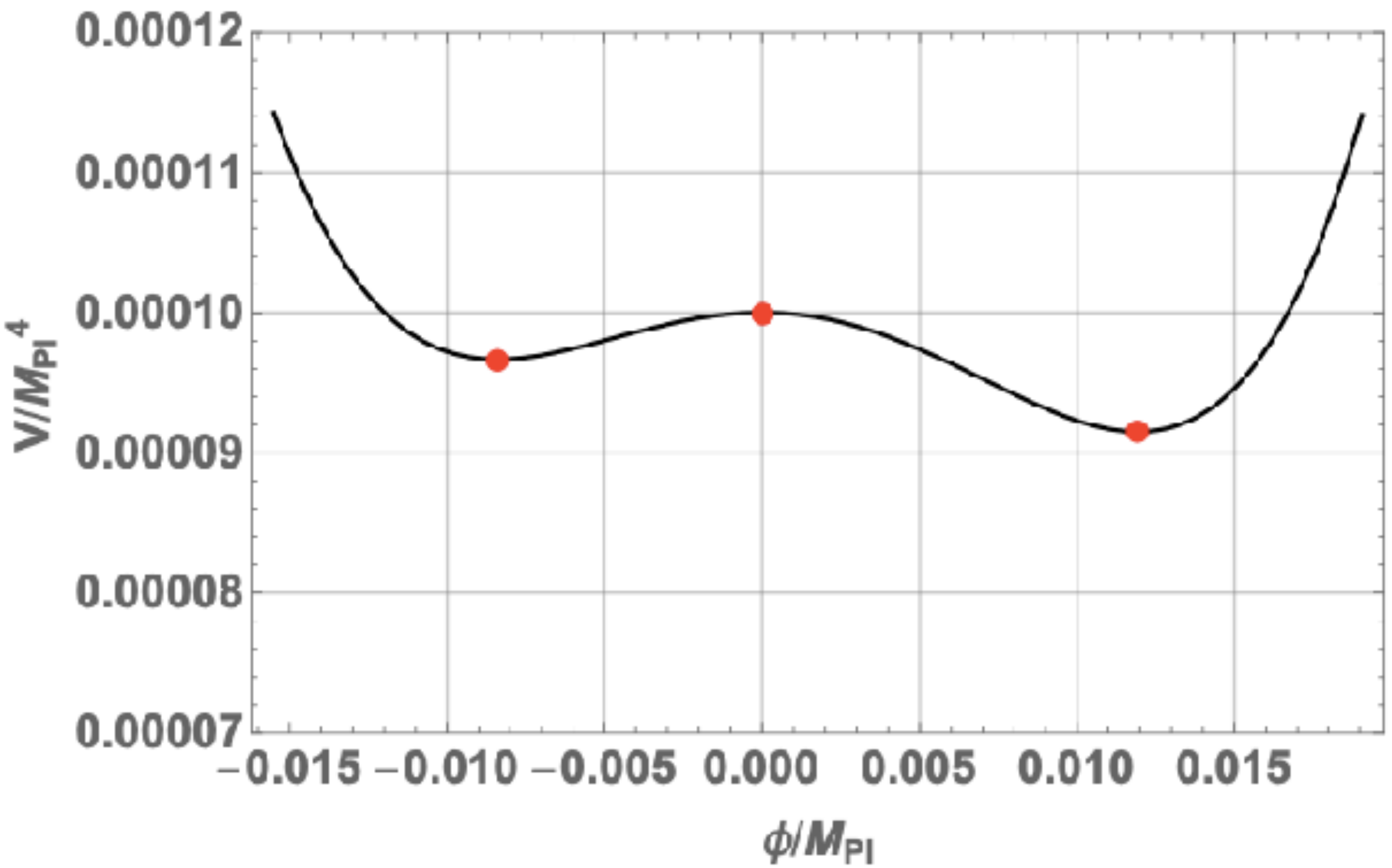
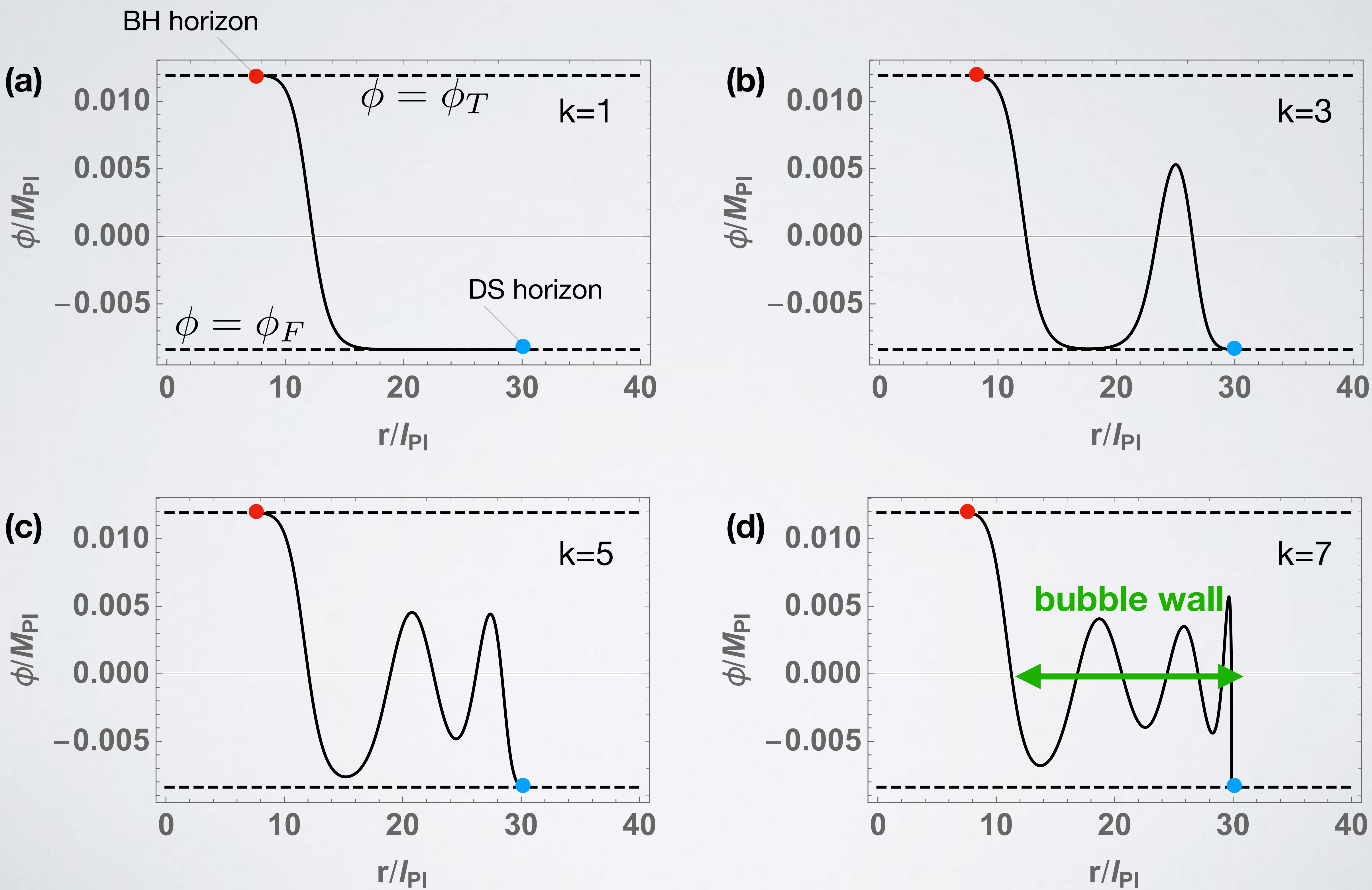
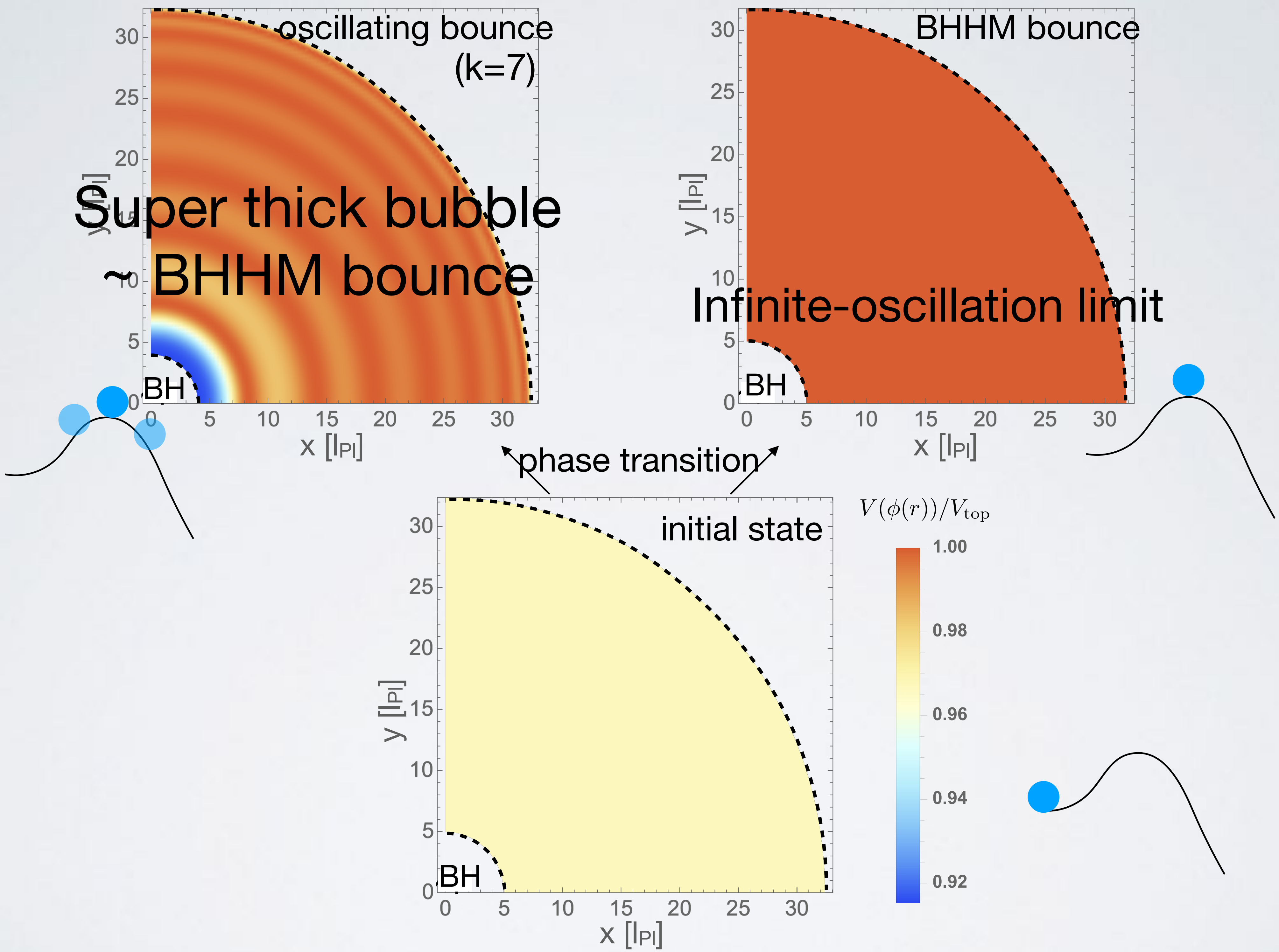


Figure 3. Plot of the effective potential $V(\phi)$ with $V_{\rm top} = 10^{-4} M_{\rm Pl}^4$, $\beta = 250$, $g = 1/\sqrt{8}$, and $v = 0.01 M_{\rm Pl}$. Red points show the false vacuum, true vacuum, and the top of barrier.

RESULTS

R. Gregory, I. G. Moss, NO (2020)

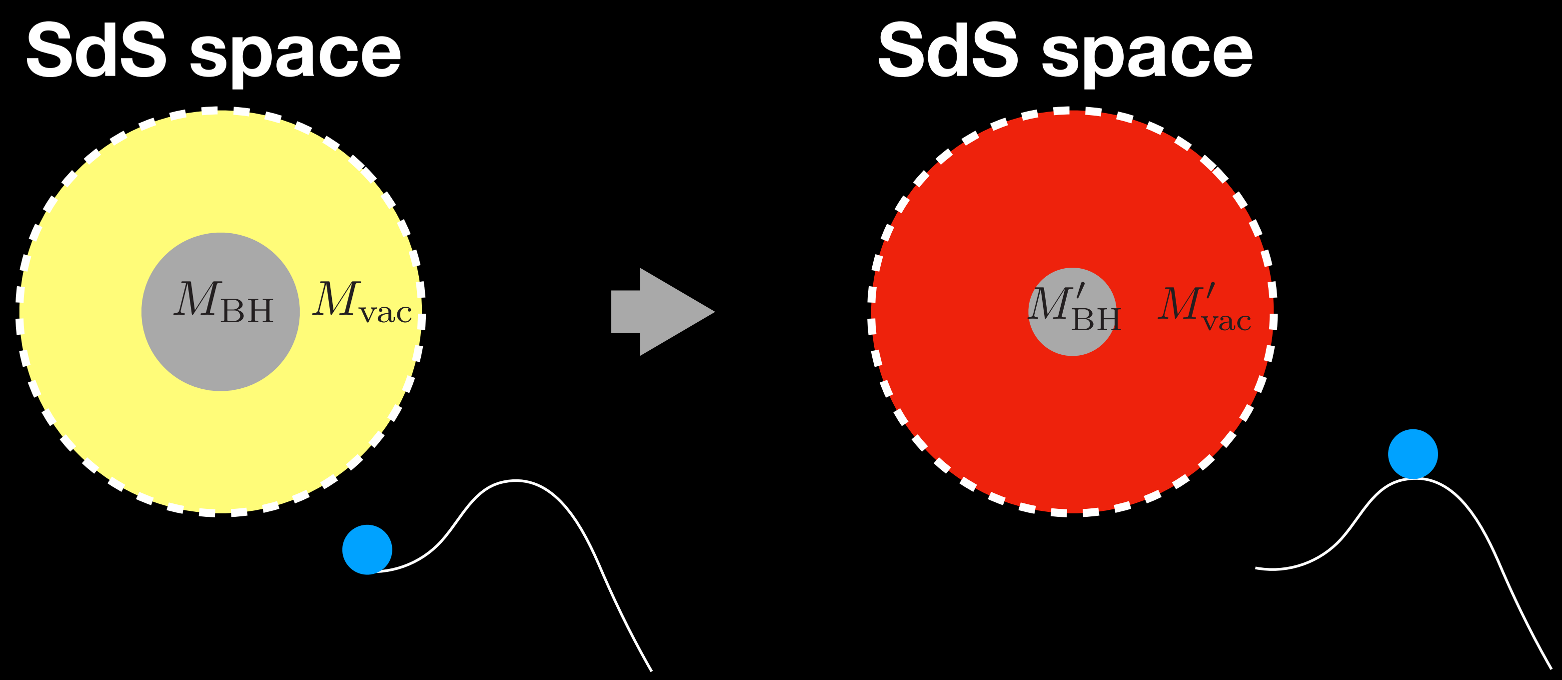




oscillating bounce
(k=7)
Super thick bubble
~ BHHM bounce
BHHM bounce
Infinite-oscillation limit
phase transition
initial state
BH
x [lPl]
y [lPl]
$V(\phi(r))/V_{\rm top}$
1.00
0.98
0.96
0.94
0.92

BHHM bounce

Transition between two SdS spacetimes



Conservation of the total energy inside the cosmological horizon

$$M_{\rm BH} + M_{\rm vac} = M'_{\rm BH} + M'_{\rm vac}$$

HM vs BHHM

$$\Gamma_{\rm HM} \sim e^{-\Delta S_{\rm E}} = e^{\Delta A_{\rm c}/4G} \simeq e^{-\Delta E/T_{\rm dS}}$$

increment of the internal energy → exponential suppression

In our case, there exists not only vacuum energy but also a seed BH.

→ vacuum can consume the energy of BH to go to the potential top!

conservation of the internal energy inside the cosmological horizon

Gregory, Moss, NO, Patrick (2020)
arXiv: 2007.11428

||

entropy before HM transition $S_c = S'_c$ entropy after HM transition

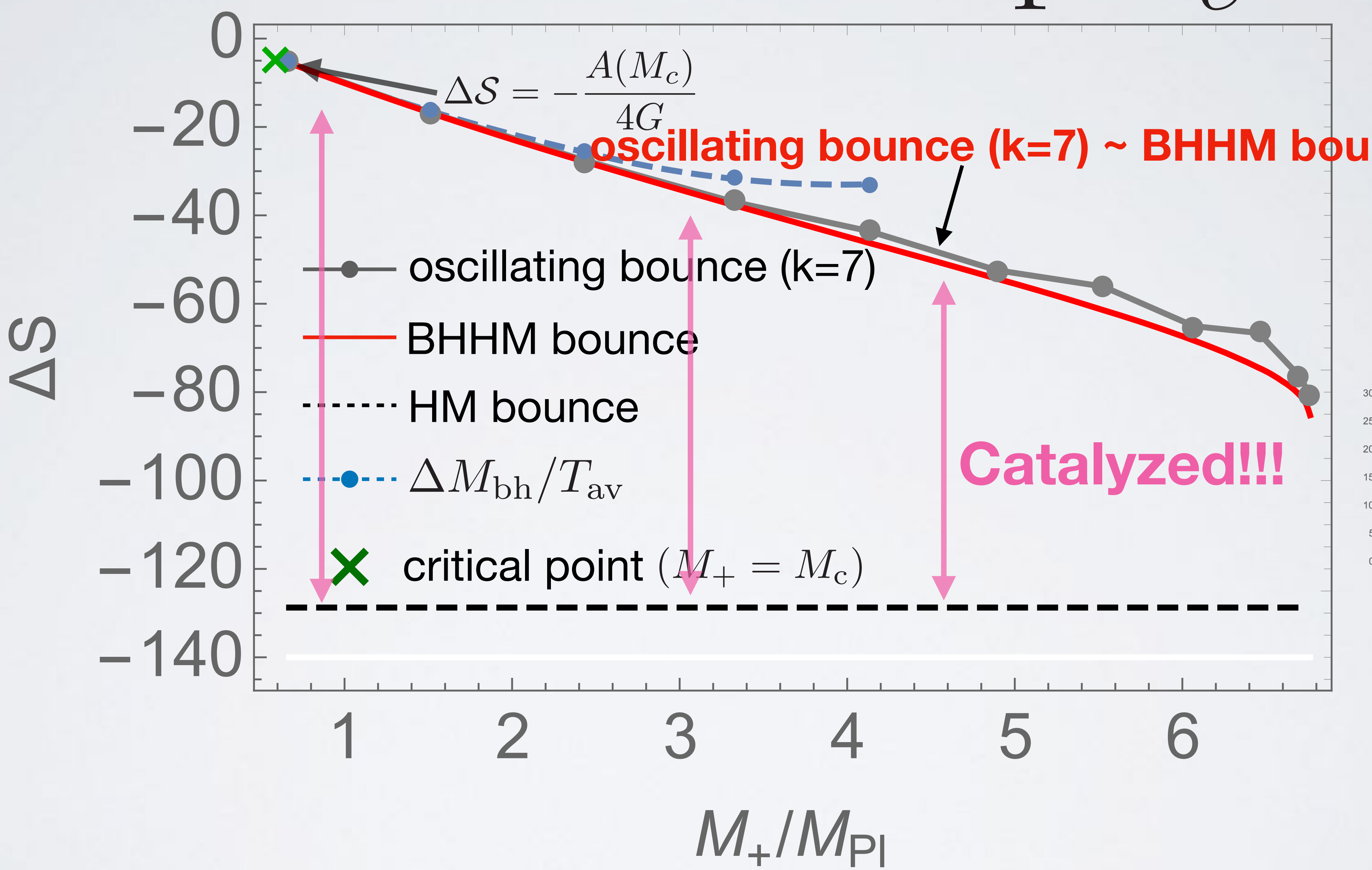
||

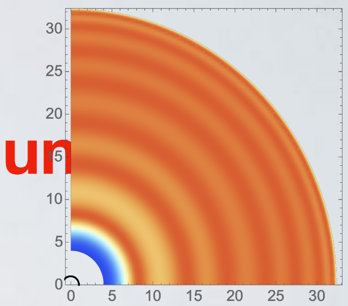
Radius of the cosmological horizon before HM transition $r_c = r'_c$ Radius of the cosmological horizon after HM transition

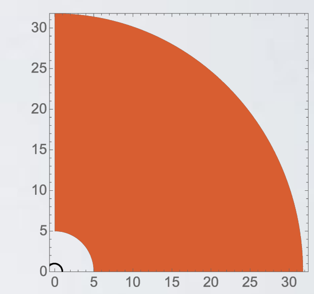
$$\Gamma_{\rm BHHM} \sim e^{-\Delta S_{\rm E}} = e^{\frac{(\Delta A_c + \Delta A_{BH})}{4G}} = e^{\frac{\Delta A_{BH}}{4G}}$$

EUCLIDEAN ACTION -COMPARISON-

$$\Gamma \sim e^{\Delta S}$$







Issues

- Vacuum decay with gravity has many issues such as the negative mode problem, instability of Euclidean action, not sure if the O(4) symmetry gives the most probable decay process.
- Can a vacuum decay process with a seed black hole be formulated with the Lorentzian path integral?
- AdS/CFT -> CDL in AdS / CFT (What if vacuum decay happens in AdS-BH?) J. L. F. Barron et al. (2010)
- Does the most probable decay have O(3) symmetric solution for a BH vacuum decay?
- BH-HM bounce can be consistent with the Fokker-Planck computation?